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On controlling current distribution in superconducting cables

Miguel Angel Moreles *, Francisco Canales

CIMAT, Jalisco S/N, Valenciana, Guanajuato, GTO 36240, Mexico

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Abstract

We present a revised version of a polynomial system modelling current distribution in a superconducting power cable. We show that by using the eigenvalue theorem in Algebraic Geometry, a numerical method can be developed to design a superconducting cable satisfying a predetermined current distribution.

Keywords: Homogeneous current distribution; Superconducting cables; Winding direction; Winding pitch

1. Introduction

There is a widespread belief that superconductivity is going to be a vital 21st century technology, not just in the power applications field but also in electronics. It is reflected on the extensive literature reporting on modelling, simulation and testing of superconducting devices. In particular, for electric power applications see the review in [7].

The interest in applying superconductivity to electric power and energy storage applications, is directly related to expectations for improved performance and efficiency advantages over conventional devices. In the case of superconducting cables, attractive is the larger amount of current and energy that can be transferred using superconductors compared to copper cables, and the energy savings that can be obtained with the superconductor. The superconducting cable is the object of study in this work.

Let us describe the device. A high temperature superconductor (HTS) power transmission cable is usually made of several layers of helically wound superconductor tapes. The current distribution among the conductor tapes is controlled mainly by pitches and winding directions of the layers, because the inductance of the layer is determined by the pitch and the winding direction.

As quoted in [9], *One of the most serious problems of this multi-layer alignment is non-uniform current distribution among the layers. If the layers do not share the current evenly, current capacity of the whole cable is much less than expected by critical currents of the conductors and the number of conductors.* Thus, the point of research, is to find efficient configurations, pitches and winding directions of the layers in order to satisfy the homogenous current condition. A relevant step in that direction is to propose and analyze mathematical models associated to the problem.

In [6], a generalized equation of the current distribution under the external field is introduced, and consequently the current distribution equation under a self-field only is derived. Therein all physical aspects are fully explained, we shall focus on the mathematical and numerical aspects of the model.

For a cable of m layers the following system is obtained:

$$\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)}\sum_{i=1}^{k} I_i + \left(\frac{\varepsilon_k}{L_k}-\frac{\varepsilon_{k+1}}{L_{k+1}}\right)\sum_{i=1}^{k}\pi r_i^2\left(\frac{\varepsilon_i}{L_i}I_i\right) + \cdots + \left(\frac{\varepsilon_k}{L_k}\pi r_k^2-\frac{\varepsilon_{k+1}}{L_{k+1}}\pi r_{k+1}^2\right)\sum_{i=k+1}^{m}\left(\frac{\varepsilon_i}{L_i}I_i\right)=0,$$

$$k=1,2,\ldots,m-1. \tag{1}$$

* Corresponding author.
E-mail address: moreles@cimat.mx (M.A. Moreles).

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The parameters are:

- I_i: current in layer i.
- r_i: radius of layer i.
- ε_i: winding direction in layer i. $\varepsilon_i = -1, +1$ depending on the twist direction in the layer, Z or S.
- L_i: twist pitch in layer i.

A cable configuration is a set of values for ε_i, L_i. If a configuration and the radii are given, we can determine the current distribution in the cable combining the total current condition as follows:

$$\sum_{i=1}^{m} I_i = I_T. \tag{2}$$

For a desired total transport current I_T, we have a linear system for currents I_i. To obtain physical solutions, parameters are to be chosen appropriately.

The problem of homogeneous current distribution can be formulated as follows: Assume that a cable of m layers is to be built with known radii r_i, $i = 1, 2, \ldots, m$. Find configurations satisfying (1) with $I_i = I_T/m$, $i = 1, 2, \ldots, m$.

Hence the interest is to solve the system

$$k\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)} + \left(\frac{\varepsilon_k}{L_k} - \frac{\varepsilon_{k+1}}{L_{k+1}}\right)\sum_{i=1}^{k}\pi r_i^2\left(\frac{\varepsilon_i}{L_i}\right) + \cdots + \left(\frac{\varepsilon_k}{L_k}\pi r_k^2 - \frac{\varepsilon_{k+1}}{L_{k+1}}\pi r_{k+1}^2\right)\sum_{i=k+1}^{m}\left(\frac{\varepsilon_i}{L_i}\right) = 0,$$
$$k = 1, 2, \ldots, m-1, \tag{3}$$

for ε_i, L_i, $i = 1, 2, \ldots, m$.

We see that system (3) is quadratic on the variables ε_i/L_i. Also, it is known that in actual cables the number of layers is small, four or six are the most common. Thus, symbolic algorithms for solving polynomial systems are suitable for solutions. Some of these algorithms are based on techniques from Algebraic Geometry. In this work, we use the eigenvalue theorem in this theory to develop a hybrid algorithm, symbolic-numeric, to design a superconducting cable satisfying a predetermined current distribution. The algorithm rests on the theory of Gröbner bases. The content is as follows.

In Section 2 we present a slight modification of system (3) for modeling current distribution. A general algorithm for solving polynomial systems is sketched in Section 3. Also, the algorithm to solve the system of current distribution is presented. In Section 4 we provide some numerical examples of the applicability of the algorithm. More importantly, we report on actual cables whose current distributions correspond to the ones predicted by the algorithm. In Section 5 we comment on our work and future research.

2. Current distribution in superconducting cables

Instead of uniform current distribution we consider a more general situation. In normalized form, Eq. (2) reads

$$\sum_{i=1}^{m} I_i = 1. \tag{4}$$

In actual situations perfect efficiency is seldomly attained. Thus we introduce the efficiency equation

$$\sum_{i=1}^{m} E_i = E, \tag{5}$$

where $0 < E \leqslant 1$. Here E_i denotes the efficiency in layer i, $0 < E_i \leqslant 1/m$.

System (3) becomes

$$\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)}\sum_{i=1}^{k}E_i + \left(\frac{\varepsilon_k}{L_k} - \frac{\varepsilon_{k+1}}{L_{k+1}}\right)\sum_{i=1}^{k}\pi r_i^2\left(\frac{\varepsilon_i}{L_i}E_i\right) + \cdots + \left(\frac{\varepsilon_k}{L_k}\pi r_k^2 - \frac{\varepsilon_{k+1}}{L_{k+1}}\pi r_{k+1}^2\right)\sum_{i=k+1}^{m}\left(\frac{\varepsilon_i}{L_i}E_i\right) = 0,$$
$$k = 1, 2, \ldots, m-1. \tag{6}$$

Observe that in (6) the winding direction ε_i, and twist pitch L_i appear in the form ε_i/L_i. Letting $l_i = \varepsilon_i/L_i$ we obtain a quadratic system of $m-1$ equations with the m unknowns $l_1, l_2, \ldots, l_m$. Namely

$$\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)}\sum_{i=1}^{k}E_i + (l_k - l_{k+1})\sum_{i=1}^{k}\pi r_i(l_iE_i) + \cdots + (l_k\pi r_k^2 - l_{k+1}\pi r_{k+1}^2)\sum_{i=k+1}^{m}(l_iE_i) = 0,$$
$$k = 1, 2, \ldots, m-1. \tag{7}$$

In the sections that follow, we shall present an algorithm to solve this system.

Remarks

(i) There are classical numerical techniques available to solve this polynomial system, but as quoted in [11], *there are no good, general solvers for solving systems of multivariate polynomial equations*. For systems of moderate size, symbolic algorithms ought to be considered. These algorithms reduce the problem to compute roots of a univariate problem, or to solve eigenvalue problems.

(ii) There is a great deal of experimental work on HTS cables. In [2] a report is presented on cables of 4, 8, and 10 layers which were built and tested. In [10], experiments were carried out in cables of 2, 4 and 10 layers also. In both cases current distribution is not uniform. The condition of (almost) homogeneous distribution is satisfied in cables reported in [12] (four layers) and [8] (six layers).

3. Solving equations via eigenvalues

The solution of the system (7) will be reduced to eigenvalue problems. The method is based on a well known theorem in Algebraic Geometry. A leisure introduction is presented in [1].

3.1. The eigenvalue theorem

Let us introduce the basics to state the result. Let $\mathbb{C}[x_1,x_2,\ldots,x_n]$ be the ring of polynomials in n variables with complex coefficients. Let $f_i \in \mathbb{C}[x_1,x_2,\ldots x_n]$, $i=1,2,\ldots,s$. Consider the system of equations

$$\begin{aligned} f_1 &= 0,\\ &\vdots \\ f_s &= 0. \end{aligned} \tag{8}$$

Let $I=\langle f_1,\ldots,f_s\rangle$ be the ideal generated by $f_1,\ldots,f_s$, and let $\mathbf{U}$ be the quotient ring $\mathbb{C}[x_1,x_2,\ldots,x_n]/I$. $\mathbf{U}$ is a vector space. Below an appropriate basis for $\mathbf{U}$, denoted by $\mathscr{B}$, is constructed. Also, given $g \in \mathbb{C}[x_1,x_2,\ldots,x_n]$, we shall define the multiplication map m_g from $\mathbf{U}$ in itself.

We have the following remarkable result.

3.1.1. Eigenvalue theorem

Assume that the set of solutions of system (8) is finite. Then, for each i, $i=1,2,\ldots,n$, the eigenvalues of m_{x_i} are the x_i – coordinates of the solutions of system (8).

This theorem does not tell us how to match up the various coordinates. One can do this by a variety of methods, for our application a brute force approach will suffice.

3.2. An algorithm for solving polynomial systems

The algorithm is based on the theory of Gröbner bases for ideals of polynomials. The basic steps are:

1. Compute a Gröbner basis $G=\{g_1,\ldots,g_t\}$ for the ideal $I=\langle f_1,\ldots,f_s\rangle$.
2. Compute the basis $\mathscr{B}$ of the vector space $\mathbf{U}$.
3. Find the generalized companion matrices M_{x_i} associated to m_{x_i}.
4. Compute the corresponding eigenvalues and apply the test given in the eigenvalue theorem to find the solutions of the system.

Remarks

(i) The algorithm is implemented in Maple. For steps 1–3 apparently there is not much to do, Maple provides routines to compute G, $\mathscr{B}$, M_{x_i}. In practice, what we compute in step 1 is a *Reduced Gröbner Basis*.

(ii) We observe that steps 1–3 use symbolic algorithms, whereas in step 4 the algorithm is numerical. Thus, special care is needed in the implementation.

(iii) The algorithm applies only if the set of solutions is finite. We have implemented an algorithm based on the Radical Theorem to test for this condition.

3.3. Algorithm for control of current distribution

Here we provide details on the actual implementation of the Algorithm.

Step 1. *The polynomial system*

Recall the system

$$\begin{aligned} &\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)}\sum_{i=1}^{k}E_i + t(l_k-l_{k+1})\sum_{i=1}^{k}\pi r_i^2(l_iE_i)\\ &\quad+\cdots+(l_k\pi r_k^2 - l_{k+1}\pi r_{k+1}^2)\sum_{i=k+1}^{m}(l_iE_i)=0,\\ &k=1,2,\ldots,m-1. \end{aligned} \tag{9}$$

The system is underdetermined, there are $m-1$ equations and m unknowns. For some layer, say i_0, we assume that ε_{i_0} and L_{i_0} are known. Thus, we append the equation

$$l_{i_0} - \frac{\varepsilon_{i_o}}{L_{i_0}} = 0. \tag{10}$$

Providing ε_{i_0} and L_{i_0} is not so restrictive, in most algorithms an initial guess is required.

Step 2. *Preconditioning to find a Gröbner basis*

There are many algorithms to find Gröbner bases. Most, if not all, are based on the Buchberger's Algorithm. Roughly speaking, two main ingredients of this algorithm are: (i) a generalized version of the division algorithm for polynomials and (ii) Gaussian elimination like operations. Even for small systems, computation of a Gröbner basis is lengthy and costly. Moreover, a naive application of the algorithm above may lead to ill posedness. The latter is a more serious problem. To correct it we precondition the original system (6) by means of the change of variables

$$l_i = \frac{\pi r_i^2}{L_i}. \tag{11}$$

Consequently, we compute a reduced Gröbner basis associated to the system

$$\begin{aligned} &\frac{r_{k+1}-r_k}{\pi(r_{k+1}+r_k)}\sum_{i=1}^{k}E_i + \left(\frac{\varepsilon_k l_k}{\pi r_k^2}-\frac{\varepsilon_{k+1}l_{k+1}}{\pi r_{k+1}^2}\right)\sum_{i=1}^{k}(\varepsilon_i l_i E_i)\\ &\quad+\cdots+(\varepsilon_k l_k - \varepsilon_{k+1}l_{k+1})\sum_{i=k+1}^{m}\left(\frac{\varepsilon_i l_i E_i}{\pi r_i^2}\right)=0,\\ &k=1,2,\ldots,m-1. \end{aligned} \tag{12}$$

$$l_{i_0} - \frac{\pi r_{i_0}^2}{L_{i_0}} = 0.$$

The algorithm to compute the Gröbner basis is symbolic and works with rational numbers. The float point numbers representing our real data are converted to (symbolic) rationals.

Step 3. *Companion matrices*
This part is entirely symbolic. The base $\mathscr{B}$ of $\mathbb{C}[x_1, x_2, \ldots, x_n]/I$ is computed, as well as the generalized companion matrices.

Step 4. *Solutions*
The coordinates of the companion matrices are converted to floating point numbers. Then eigenvalues are found. Finally, we select those solutions so that the twist pitches are within a desired range.

3.3.1. Global strategies

This algorithm may be regarded as local, a layer i_0 is selected and an *educated* initial guess for ε_{i_0} and L_{i_0} is provided. For people working in the manufacturing process this might be enough. Fortunately, it is straightforward to develop global strategies for improvement. For instance, in step 1 we may prescribe all twist directions and carry out the algorithm for L_{i_0} in a desired range. More generally, an outer loop may run over (all possible) sets of twist directions. In the former case we may try the following two combinations, see [12]:

(i) All layers are twisted in one direction (so called "one direction twist" or ODT).
(ii) Part of layers (usually one half) is twisted in one direction and the rest in another one. The twist direction is changed only once. It is so called "two direction twist" or TDT.

4. Numerical and physical validation

In Table 1 we list the radii of the layers of a cable considered in [6].

Let us consider the TDT and sweep L_1 in the interval [50, 1000]. In Fig. 1 we plot the relation between the first layer twist pitch and the others for the homogeneous current distribution. This is exactly Fig. 5b in [6].

This example illustrates that not all twist pitches are practical, or even constructible. Indeed, it is observed that the second, the third and the fourth layer pitches are extremely short compared with the first layer. The pitch for the first layer is about 1000 mm whereas the others are in the range of 15–45 mm. As noted in [6], this fact makes the fabrication of the cable difficult because of its large bending strain. Also, if the pitch of a layer is too large, which is the case of the first layer, it will be hard to assemble.

Table 1
Radii of coaxial layers

Layer	1	2	3	4
Radius r (mm)	10.0	10.5	11.0	11.5

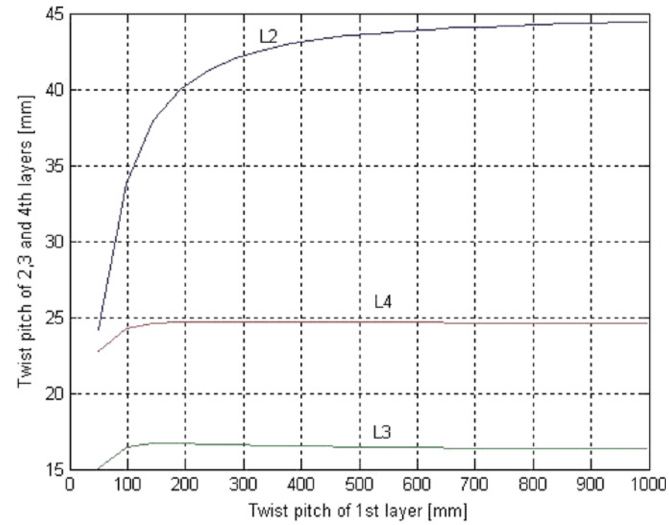

Fig. 1. Twist pitches of second, third and fourth layers as a function of that of the first layer for homogeneous current distribution in case of alternative twist directions. Radii as in Table 1.

In applications an appropriate range for pitches is between 150 mm and 900 mm. Also, for real data, radii are between the range of 10 mm and 30 mm. Except for the last example, we shall provide examples within these ranges.

4.1. Numerical examples

Let us consider the ODT in the example above. In Fig. 2 we reproduce easily Fig. 4c in [6].

For a four layer cable the original system can be reduced to quadratic equations. Although not advisable, radical expressions are possible for the solutions. For five and six

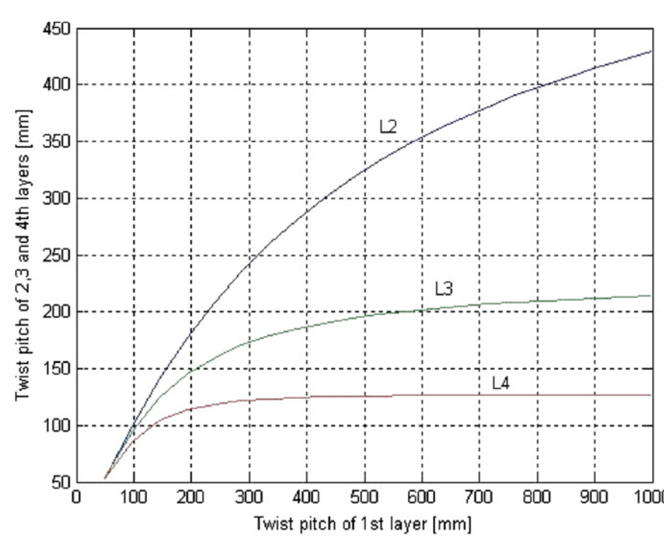

Fig. 2. Twist pitches of other layers as a function of that of the first layer for homogeneous current distribution in the ODT case. Radii as in Table 1.

layers we have non-trivial applications of our hybrid algorithm. To illustrate, we show examples of constructible cables with homogenous current distributions.

In Table 2 we list the radii of a five layers cable in [6]. In Fig. 3 therein, it is shown that the superconductor with same pitch (320 mm) and ODT has imbalanced current distribution. With these radii and one direction twist, our algorithm yields the pitches in Fig. 3 for homogeneous current distribution. Let us choose a cable with a pitch of 320 mm in layer 1, the full configuration is listed in Table 3. Compare with Table 1 in [6].

Let us add a sixth layer. In Table 4 we show a potential cable with homogeneous current distribution and the full results are shown in Fig. 4.

4.2. Physical validation

At CIDEC (CONDUMEX Group, Mexico), a cryogenic laboratory has been built to manufacture and test HTS cables. In particular, the four and six layer cables reported in [12,8]. The radii and direction twist of these cables are listed in Tables 5 and 6.

Table 2
Radii of coaxial layers

Layer	1	2	3	4	5
Radius r (mm)	21.28	21.46	21.64	21.82	22.00

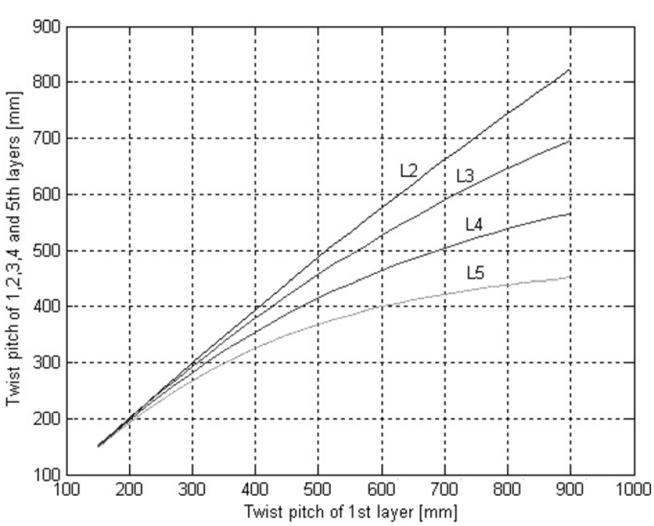

Fig. 3. Twist pitches of other layers as a function of that of the first layer for homogeneous current distribution in the ODT case. Radii as in Table 3.

Table 3
Pitches for homogeneous current distribution, $L_1 = 320.00$ mm given

Layer	1	2	3	4	5
Radius r (mm)	21.28	21.46	21.64	21.82	22.00
Twist direction	1	1	1	1	1
Pitch L (mm)	320.00	318.55	310.75	297.66	280.61

Table 4
Pitches for homogeneous current distribution, $L_1 = 320.00$ mm given

Layer	1	2	3	4	5	6
Radius r (mm)	21.28	21.46	21.64	21.82	22.00	22.18
Twist direction	1	1	1	1	1	1
Pitch L (mm)	320.00	319.72	314.09	303.83	289.89	273.31

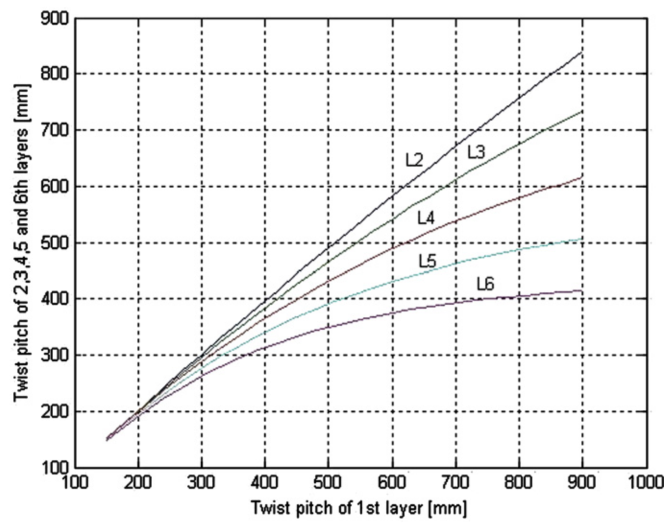

Fig. 4. Twist pitches of other layers as a function of that of the first layer for homogeneous current distribution in the ODT case. Radii as in Table 4.

Table 5
Configuration of cable

Layer	1	2	3	4
Radius r (mm)	25.55	25.84	26.18	26.54
Twist direction	1	1	−1	−1

Table 6
Configuration of cable

Layer	1	2	3	4	5	6
Radius r (mm)	27.53	27.71	28.1	28.56	28.8	29.1
Twist direction	1	1	1	−1	−1	−1

Pitches for homogeneous current distribution are shown in Figs. 5 and 6.

The ultimate test of a numerical model is its capability to reproduce laboratory results. The first author worked as a consultant for CIDEC on a project to design superconducting cables. Our access to undisclosed data shows that the pitches predicted by the numerical solution correspond remarkably well with the cables built at CIDEC.

5. Final comments

System (3) is by no means the only model for numerical simulation of current distribution in superconducting cables. A finite element approach is presented in [5]. In [2,3] the cable is modeled as a nonlinear parallel circuit, a

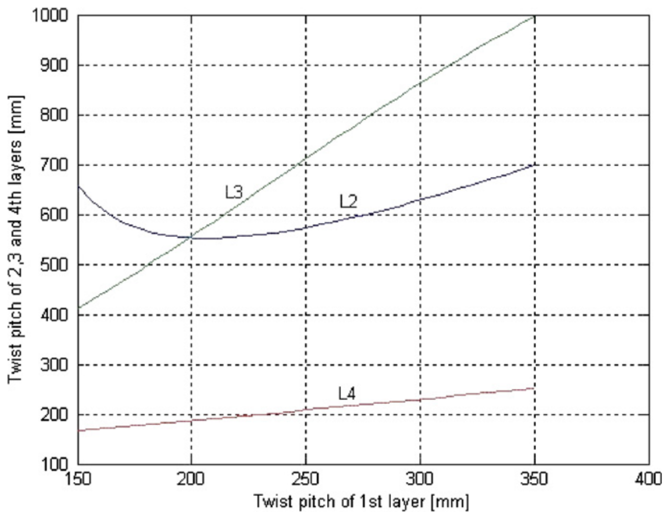

Fig. 5. Twist pitches of other layers as a function of that of the first layer for homogeneous current distribution in the TDT case. Radii as in Table 5.

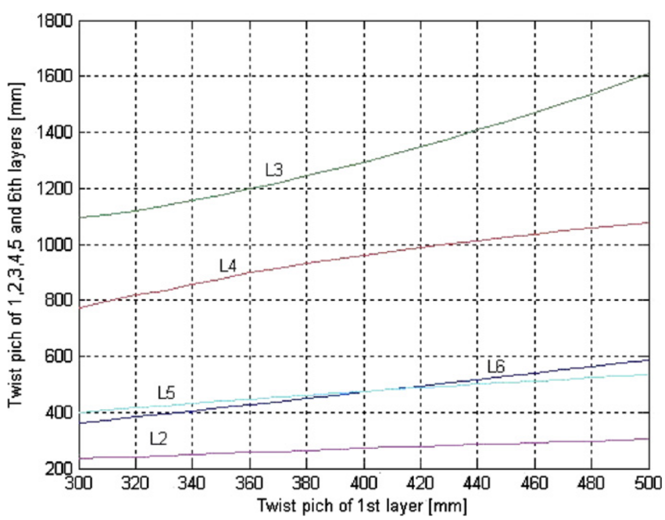

Fig. 6. Twist pitches of other layers as a function of that of the first layer for homogeneous current distribution in the TDT case. Radii as in Table 6.

system of nonlinear ODE is obtained. We have used the simpler model of Hamajima et al. [6]. Therein, an argument is given to illustrate that the algebraic system arises from the circuit model.

In some cases it is not possible to find configurations satisfying the homogeneous current condition. Since layers saturate from the outermost layer, we may reduce the efficiency in the inner layers to find a suitable design. Hence we believe it is more appropriate to pose the problem as the solution of system (6).

Our experimental investigations suggest that the algebraic model is a convenient starting point for design. Then it may be complemented with more sophisticated models involving the physical properties of the tapes. In any case, the effectivity of such a simple mathematical model is surprising. Experts can attest to the complexity of the manufacturing process.

Also, we have used the simplest tools from Algebraic Geometry. We are interested in exploring other possibilities, see [4].

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References

[1] Cox DA. Introduction to Gröbner bases. In: Cox DA, Sturnmfels B, editors. Applications of computational algebraic geometry. Proceedings of symposia in applied mathematics, vol. 53. Providence, Rhode Island: AMS; 1998.
[2] Däumling M. A model for the current distribution and ac losses in superconducting multi-layer power cables. Cryogenics 1999;39:759–65.
[3] Däumling M. Electromagnetic behaviour of a superconducting cable. IEEE Trans Appl Superconduct 2001;11(1).
[4] Dickenstein A, Emiris IZ, editors. Solving polynomials equations, foundations, algorithms, and applications. Berlin: Springer; 2005.
[5] Grilli F, Stavrev S, Dutoit B, Spreafico S. Numerical modeling of a HTS cable. IEEE Trans Appl Superconduct 2003;13(2).
[6] Hamajima T, Alamgir AKM, harada N, Tsuda M, Ono M, Takano H. Analysis of current distribution in a large superconductor. Cryogenics 2000;40:729–36.
[7] Hassenzahl WV, Hazelton DW, Johnson BK, Komarek P, Noe M, Reis CT. Electric power applications of superconductivity. Proc IEEE 2004;92(10).
[8] Jergel M, Morales A, Falcony C, Sytnikov V, Dolgosheev P, Belyi DI, et al. Superconducting transmission 23 kV/2kA cable – first in Latin America. Rev Mex Fis 2003;49(6):489–92.
[9] Kim S-W, Joo J-H, Cho J, Bae JH, Kim HJ, Seong KC. Effect of winding direction od four-layer HTS power transmission cable. Cryogenics 2003;43:629–35.
[10] Lue JW, Demko JA, Dressner L, Hughey RL, Sinha U, Tobert JC, et al. AC losses of prototype HTS transmission cables. IEEE Trans Appl Superconductivity 1999;9(2).
[11] Press WH et al. Numerical recipes in C. 2nd ed. Cambridge: Cambridge University Press; 1992.
[12] Sytnikov VE, Dolgosheev PI, Soloviev MG, I Belij D, Nieto L, Perez A, et al. The current test results for two models of HTS cables on CASAT project. IEEE Trans Appl Superconduct 2003;13(2).