Local adaptable quadrature filters to demodulate single fringe patterns with closed fringes.

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Abstract: We propose a new approach to demodulate a single fringe pattern with closed fringes by using Local Adaptable Quadrature Filters (LAQF). Quadrature filters have been widely used to demodulate complete image interferograms with carrier frequency. However, in this paper, we propose the use of quadrature filters locally, assuming that the phase is locally quasimonochromatic, since quadrature filters are not capable to demodulate image interferograms with closed fringes. The idea, in this paper, is to demodulate the fringe pattern with closed fringes sequentially, using a fringe following scanning strategy. In particular we use linear *robust quadrature filters* to obtain a fast and robust demodulation method for single fringe pattern images with closed fringes. The proposed LAQF method does not require a previous fringe pattern normalization. Some tests with experimental interferograms are shown to see the performance of the method along with comparisons to its closest competitor, which is the Regularized Phase Tracker (RPT), and we will see that this method is tolerant to higher levels of noise.

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References and links

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1. Introduction

In *moiré* interferometry, as well as in other areas of optical metrology, when one is working with transient events, one can have situations where it is necessary to deal with a single fringe pattern with closed fringes. As the information of interest is phase modulated by the fringe pattern, it is necessary to apply a demodulation method able to demodulate a single fringe pattern with closed fringes.

A single fringe pattern with closed fringes is typically modeled in the following way:

$$I(x,y) = a(x,y) + b(x,y)\cos[\phi(x,y)], \tag{1}$$

where $a(x,y)$ is the background illumination and $b(x,y)$ the modulation term or contrast. The phase to be demodulated is $\phi(x,y)$. If the fringe pattern has a carrier frequency, then it can be modeled in the following way:

$$I(x,y) = a(x,y) + b(x,y)\cos[\phi(x,y) + \omega x], \tag{2}$$

where $\phi(x,y)$ is the modulating phase, and ω is the carrier frequency. In this case, we can demodulate the fringe pattern by using quadrature filters like those used with the Fourier transform method [1], and the Hilbert transform method [2]. In both cases, we obtain the wrapped modulating phase $\phi(x,y)$. However, this kind of methods fails to obtain the modulating phase when we have a fringe pattern like the one shown in (1), or in other words, when we have a single fringe pattern with closed fringes.

In this paper we are going to show a new approach to demodulate single fringe pattern images with closed fringes, using Local Adaptable Quadrature Filters (**LAQF**). In general, a quadrature filter is a band-pass filter that is zero in one half of the Fourier domain, and maps its input to a complex space. For example, if the input is a fringe pattern like the one shown in Eq. (1), the output is a complex signal whose real part is the fringe pattern itself, and the imaginary part is its quadrature. However, when we have closed fringes, the input signal quadrature, obtained with a quadrature filter, has abrupt sign changes when applied to the complete fringe pattern [2]. Hence, quadrature operators have been proposed to overcome this drawback, i.e. the Larkin's quadrature operator and the n-dimensional quadrature transform [3, 4, 5].

Since a quadrature filter is not able to recover the expected phase from a complete single fringe pattern with closed fringes, we propose the use of adaptable quadrature filters in small regions where the phase may be considered quasimonochromatic. One of the advantages of using quadrature filters, is that we need only a band-pass filtering to remove the background illumination from the image interferogram. Then we use the LAQF to estimate the phase sequentially using a fringe following scanning strategy. This sequential way, used to demodulate

a single fringe pattern with closed fringes, remind us of the first proposed sequential method, called the Regularized Phase Tracker (**RPT**) [6], with the difference that we use local quadrature filters to estimate the phase.

As we are going to compare the LAQF method, developed here, with the RPT method shown in [6], let us to show, in brief, the use of the RPT method to demodulate single fringe pattern images with closed fringes. Also, we are going to comment the derived RPT methods that have been published recently.

1.1. RPT method

To demodulate the phase, using the RPT method, we minimize the following functional with respect to $\hat{\phi}(x,y)$ and frequencies $u(x,y)$ and $v(x,y)$:

$$U[\hat{\phi}(x,y),u(x,y),v(x,y)] = \sum_{(\eta,\xi)\in\Gamma} \left\{ [\cos p(\eta,\xi) - I'(\eta,\xi)]^2 + \lambda[\hat{\phi}(\eta,\xi) - p(\eta,\xi)]^2 \right\}, \quad (3)$$

where $p(\eta,\xi) = \hat{\phi}(x,y) + u(x,y)(x-\eta) + v(x,y)(y-\xi)$ is a phase plane and $\hat{\phi}(x,y)$ is the estimated phase at site (x,y) after the minimization. The closed region Γ is a neighborhood around site (x,y), and λ is a regularization parameter to strengthen the method against noise. $I'(x,y)$ is the fringe pattern shown in Eq. (1), but normalized in the following way:

$$I'(x,y) = \cos[\phi(x,y)] \quad (4)$$

where background illumination $a(x,y)$ is removed using a high-pass filter, and the modulation term $b(x,y)$ is spatially normalized to the constant value 1, using normalization techniques for fringe patterns [7, 8, 9]. Then, the RPT method needs a previous fringe pattern normalization to remove the contrast variations of the image interferogram.

To obtain the expected phase using the RPT, it is necessary to use a scanning strategy to follow the fringes. This scanning strategy first visits the sites from the fringe pattern that are in the same isophase contour, i.e. following the fringes. One can find an algorithm with this feature for the scanning strategy in Ref. [10]. As our approach also needs this scanning strategy to obtain the expected phase, we will refer to this scanning strategy as the Fringe Following Scanning (**FFS**) throughout this paper.

Then, to demodulate an experimental interferogram with closed fringes, using the RPT, we do the following main steps in the demodulation process:

1. Apply a band-pass filter to remove the background illumination and attenuate the noise.

2. Normalize the fringe pattern in order to make the contrast component, $b(x,y)$, spatially constant.

3. Demodulate the fringe pattern using the RPT.

In step 1, band-pass filtering may be easily achieved using low-pass filtering followed by high-pass filtering. For example, the following spatial filter may be used to apply a band-pass filter:

$$g_\sigma(x,y) = \exp\left[\frac{-(x^2+y^2)}{\sigma^2}\right], \quad (5)$$

where σ controls the filter band-width. Having this, we can apply a band-pass filter in the following way:

$$I'(x,y) = [I - I * g_{\sigma_H}(x,y)] * g_{\sigma_L}(x,y), \quad (6)$$

where $*$ is the convolution operator, $I(x,y)$ is the image interferogram modeled in (1), and $\sigma_L \ll \sigma_H$, i.e, $\sigma_L = 2.4$ and $\sigma_H = 80$, for 256×256 image interferograms or biggers. This process can be implemented in a fast way by using the fast Fourier transform [11].

Step 2 is more complicated, since it is necessary to obtain a pre-estimated phase map to normalize the fringe pattern (see [7, 8, 9]), i.e, the simplest way to normalize a single fringe pattern with closed fringes may be using the Fourier transform method reported in [1], however, this technique may introduce undesired artifacts in the fringe pattern. For this reason, to estimate the phase of experimental interferograms, the RPT method spends most of the computational time in the normalization rather than in the demodulation process itself.

Other known approaches to demodulate single fringe patterns with closed fringes that need a previous fringe pattern normalization are the Larkin's quadrature operator [3, 4], and the n-dimensional quadrature transform [5]. These methods need a normalized fringe pattern to estimate the fringe orientation angle.

As the fringe pattern normalization may on occasions turn out to be as difficult as obtaining the phase, in Ref. [12] another version of the RPT method was proposed to deal with the modulation term. Then, the modified RPT functional, proposed in [12], may be written as the following:

$$
\begin{aligned}
U[\hat{\phi}, u, v, \hat{b}, b_x, b_y] &= \sum_{(\eta, \xi) \in \Gamma} \left\{ [\beta(\eta, \xi) \cos p(\eta, \xi) - I'(\eta, \xi)]^2 \right. \\
&+ \lambda [\hat{\phi}(\eta, \xi) - p(\eta, \xi)]^2 \\
&+ \left. \mu [\hat{b}(\eta, \xi) - \beta(\eta, \xi)]^2 \right\},
\end{aligned}
\tag{7}
$$

where $\hat{b}$ is the modulation estimation, and b_x and b_y its partial derivatives in x and y respectively. We have removed the (x,y) dependence of each variable for more clarity. The term $\beta(\eta, \xi)$ is a modulation plane defined as $\beta(\eta, \xi) = \hat{b}(x,y) + b_x(x,y)(x - \eta) + b_y(x,y)(y - \xi)$. If we see Eq. (3) and Eq. (7), we can see that the modified RPT method reported in [12], estimates the phase and frequencies along with the modulation term and its derivatives. As a consequence, using the modified RPT method reported in [12], let us solve a non-linear system with more time-consuming numerical methods, than the numerical methods used to solve the non-linear system of the original RPT reported in [6].

More recently, in Ref. [13], a half-quadratic linearized RPT functional was proposed for the phase estimation that let us solve a linear system instead of the original RPT non-linear system. Taking this idea, and the idea shown in [12] to estimate the phase and modulation term, in Ref. [14] a linearized modified RPT method is presented to estimate the phase and modulation term by solving a linear system. The main advantage in this work is that one can have a linearized RPT method that estimates the phase and modulation term in the same process as using Eq. (7), but much faster (see [14]).

All versions of the RPT method, mentioned in the last paragraph, model the phase estimation and the modulation or contrast component of the image interferogram, by using a cost functional like the shown in Eq. (7). As a result, these modified RPT methods are more robust than the original RPT that uses Eq. (3), when the image interferogram is not normalized. However, if we see Eq. (3) and Eq. (7), these RPT like methods support the same levels of noise than the original RPT, since the added terms in Eq. (7) just deal with the modulation variations.

In this paper we present the local adaptable quadrature filter method or LAQF method, which does not need a previous fringe pattern normalization like the modified RPT methods shown in [12, 13, 14], and let us solve a linear system like the method reported in [14]. As we use local quadrature filters to estimate the local phase, rather than estimate the local phase by fitting a phase plane with the RPT method, the LAQF method reported here supports higher levels of noise than the RPT like methods.

In section 2 we are going to show in detail the LAQF method, which particularly uses Robust Quadrature Filters [15] to estimate the phase locally. In section 3, we are going to present comparisons between the proposed method and the RPT to see that this method, with a simple band-pass filtering, is able to demodulate noisier fringe patterns than the RPT method. Also, we will show some experimental interferograms to see its performance with real fringes. Finally, in section 4 we will talk about the conclusions of the work.

2. Proposed LAQF method

As we said before, in this method we only use a band-pass filtering previous to the fringe pattern demodulation using the LAQF. Then, here it is not necessary to normalize the fringe pattern.

Assuming that we apply a band-pass filtering to the fringe pattern given in (1), we obtain the following fringe pattern:

$$I(x,y) = b(x,y)\cos[\phi(x,y)], \tag{8}$$

where (x,y) is a site in L, being L the lattice where the interferogram is recorded. Now, assume that in a small region Γ around site (x,y), the fringe pattern locally is quasimonochromatic. Then, locally the fringe pattern looks like:

$$I_0(\eta,\xi) = b(\eta,\xi)\cos[\phi(x,y) + u_0(x-\eta) + v_0(y-\xi)], \tag{9}$$

where $(\eta,\xi) \in \Gamma$ and (u_0,v_0) are the local frequencies in x and y respectively. As one can see in Eq. (3), the RPT method also uses this assumption and fits the observed data in Γ around (x,y) with a phase plane to estimate the phase at site (x,y) (see Eq. 3). Although this technique works properly, we have found a better and more robust way using local adaptable quadrature filters to estimate the phase sequentially.

In particular, to estimate the phase in region Γ around site (x,y), we use the Robust Quadrature Filters developed in [15], to locally demodulate the fringe pattern. However, we do not apply these filters to the complete image interferogram as are presented in [15], but we apply these filters locally in a small region Γ around a site (x,y), and we adapt its tuning frequency as we move through the image interferogram's sites using a fringe following scanning or FFS. Hence, we call this method local adaptable quadrature filters or LAQF along this paper.

Then, the LAQF uses robust quadrature filters to locally estimate the phase. This process is done by minimizing the following cost functional in region Γ around site (x,y):

$$U[f] = R_\Gamma[f,I] + \lambda V_\Gamma[f], \tag{10}$$

where the filter estimation model f is complex and can be expressed as $f(\eta,\xi) = \varphi(\eta,\xi) + i\psi(\eta,\xi)$, for $(\eta,\xi) \in \Gamma$.

The first term $R_\Gamma[f,I]$ in (10) is commonly known as the data term, and it depends on the difference between the observed data I (in this case the interferogram) and the estimation model f, in such a way that $R_\Gamma[f,I]$ is minimal when f is close to I. Here, we define $R_\Gamma[f,I]$ as the residual between the finite differences of the observed data and the finite differences of the filter estimation model in the following way:

$$R_\Gamma[f,I] = \sum_{(\eta,\xi)\in\Gamma} \|f_x(\eta,\xi) - 2I_x(\eta,\xi)\|^2 + \|f_y(\eta,\xi) - 2I_y(\eta,\xi)\|^2, \tag{11}$$

where f_x and I_x, are the finite differences in x and f_y, and I_y are the finite differences in y. For example, the finite differences for f in x may be given as $f_x(x,y) = f(x,y) - f(x-1,y)$, and so on. We define the data term in this way because if we take its Fourier transform, this term has a zero in the origin, which is a desired feature in the use of quadrature filters to demodulate fringe patterns (see [2]).

The second term $V_\Gamma[f]$ is usually called the regularization term. This term adds restrictions to the estimation model f. For example, a commonly used regularization term to restrict the filter estimation model f from being smooth (to obtain a low-pass filter), is the quadratic norm of the *Laplacian*'s operator known as the *membrane model*. In our case, since we want a quadrature filter, we define the regularization term in the following way, according to Ref. [15]:

$$V_\Gamma(f) = \sum_{(\eta,\xi)\in\Gamma} \left\{ \|f(\eta,\xi) - f(\eta-1,\xi)e^{-iu_0}\|^2 + \|f(\eta,\xi) - f(\eta,\xi-1)e^{-iv_0}\|^2 \right\}, \quad (12)$$

where $i = \sqrt{-1}$, u_0 is the tuning frequency in x, and v_0 the tuning frequency in y. In the Fourier domain, this term, along with $R_\Gamma[f,I]$, looks like a band-pass filter that is zero in one half of the Fourier domain, and maximum in frequency (u_0, v_0) (see [15]). The parameter λ in (10) is known as the regularization parameter that controls the strength of the quadrature filter (or its bandwidth).

To minimize Eq. (10), we obtain its gradient with respect to f and equal it to zero. Then, we can solve the resulting linear equation system using fast straightforward algorithms such as the *Gauss-Seidel* algorithm, although we can use more generic algorithms, like the *Steepest-descent* used by the RPT method [16, 6]. In Ref. [15], the gradient of cost functional (10) is given explicitly.

Thus, to obtain the local phase in region Γ around site (x,y), we minimize Eq. (10) with respect to f using the tuning frequency (u_0, v_0). Once given the quadrature filter f by minimizing (10), we obtain the phase (modulus 2π) in region Γ as:

$$\hat{\phi}(\eta,\xi) = \arctan\left[\frac{\psi(\eta,\xi)}{\varphi(\eta,\xi)}\right]. \quad (13)$$

Given the phase in region Γ around the current site (x,y) by using (13), we move to the next site using the fringe following scanning or FFS as with the RPT. Just before we move to the next site, however, we need to update the LAQF tuning frequency to estimate the phase correctly in region Γ around the next site. The local frequencies for current site (x,y) may be used as the tuning frequency to estimate the phase in the next site, and are obtained as the finite differences along x and y of the estimated phase given by (13). But, as we see from Eq. (13), the estimated phase in region Γ around site (x,y) is wrapped into the $[-\pi, \pi]$ interval. For this reason, we take the local frequencies for the current site (x,y) in the following way:

$$u_0 = \arctan\left[\frac{\sin[(\hat{\phi}(x,y) - \hat{\phi}(x_+,y))(x-x_+)]}{\cos[(\hat{\phi}(x,y) - \hat{\phi}(x_+,y))(x-x_+)]}\right] \quad (14)$$

and

$$v_0 = \arctan\left[\frac{\sin[(\hat{\phi}(x,y) - \hat{\phi}(x,y_+))(y-y_+)]}{\cos[(\hat{\phi}(x,y) - \hat{\phi}(x,y_+))(y-y_+)]}\right], \quad (15)$$

where x_+ and y_+ are the coordinates of the previous already phase estimated site, given by the FFS, and they may take the following values:

$$x_+ = \begin{cases} x+1 & \text{If previous site is to the right} \\ x-1 & \text{If previous site is to the left} \end{cases} \quad (16)$$

and

$$y_+ = \begin{cases} y+1 & \text{If previous site is down} \\ y-1 & \text{If previous site is up} \end{cases}, \quad (17)$$

For illustration purposes, In Fig. 2, we see an example of the estimated phase in the region Γ of size 32×32 around site $(32,32)$ from the given fringe pattern. Here, we used the tuning

frequency $(u_0, v_0) = (0.25, 0.24)$ with $\lambda = 50$ to obtain the phase in Γ. We must remark that in this case, for illustration purposes we have chosen a region size too big compared with the sizes of $n \times n$, with typically, $n \in 5, 6, 7, 8$.

The scanning strategy to demodulate all sites from the fringe pattern with the LAQF is the same FFS used by the RPT method, as we said before. We start at site (x, y) as the initial seed to demodulate the fringe pattern. Then, we visit each site from the fringe pattern using the FFS. For each visited site, we estimate its local phase by minimizing Eq. (10) and using Eq. (13). After this, we obtain the tuning frequency for the next site to visit using (14) and (15).

To estimate the phase for the initial seed (x, y), we must know previously the tuning frequency to use. Here, we propose a simple way to choose the initial seed (x, y) and its tuning frequency by using a Gabor filter. A Gabor filter is a quadrature filter defined in the following way:

$$ g_{u_0,v_0}(x,y) = \exp\left[\frac{-(x^2+y^2)}{\sigma^2}\right] e^{-i(u_0 x + v_0 y)}, \tag{18} $$

where σ controls the bandwith of the Gabor filter, and (u_0, v_0) are its tuning frequencies. Thus, we tune the Gabor filter onto a frequency (u_0, v_0) that can be in the fringe pattern. Then, we filter the fringe pattern with the tuned Gabor filter. Finally, we take as initial seed the site (x, y) for which the Gabor filter magnitude response is maximal. For example, suppose that $\tilde{I}$ is the filtered fringe pattern with the Gabor filter tuned to the frequency (u_0, v_0), then we take as initial seed the site:

$$ (x, y) = \arg \max_{(x_i, y_i)} \|\tilde{I}(x_i, y_i)\|, \tag{19} $$

then we use the tuning frequency (u_0, v_0) to start the demodulation process.

Readers may wonder why use a Gabor filter to choose the initial seed and its tuning frequency instead of the robust quadrature filter shown in Eq. (10). The answer is because in this case, we apply the Gabor filter to the complete fringe pattern, which is faster than minimizing (10) for the complete pattern. Another question could be why not to use just Gabor filters instead of the robust quadrature filters to filter the local region Γ in the demodulation process. The answer is that since the Gabor filters are convolution filters, they introduce errors on the edges of the region where the filter is applied. As the region's sizes used here are small, to ensure the validity of the assumption of local quasimonochromatic phase, errors introduced on the edges by using convolution filters are significant in these small regions. Hence, we use the robust quadrature filters locally, because these filters have no problems on the region's edges.

3. Tests and results

In all tests presented here, we demodulate the given single fringe patterns with closed fringes successfully, using the following basic steps:

1. Apply a band-pass filter to attenuate the noise and remove the background contribution.

2. Demodulate the fringe pattern using the LAQF as described in previous section.

To apply the band-pass filter, we used Eq. (6) with $\sigma_L = 2.5$ and $\sigma_H = 80$ using a fast Fourier transform algorithm [11]. The computational time spent in this process, for a 256×256 im-age insterferogram, was of 0.543 seconds. For our tests we used a 64bit personal computer architecture in a Linux like operating system, and a C-language 64bit optimized compiler.

In our first test, we show the levels of noise that the LAQF tolerates, and compare our results to those obtained with the RPT method. In Fig. 2, we see a table with the obtained results. The fringe patterns shown there were computer generated with the ground truth phase shown in Fig. 3, adding a phase noise whose probability distribution is Gaussian with mean zero and

variance given in the variance column. Here, the fringe patterns are already normalized in order to be used with the RPT directly. The parameters used with the RPT method are $\lambda = 20$ and neighborhood size 6×6. The parameters used with the LAQF are $\lambda = 5$, neighborhood Γ size 6×6.

The computational cost time in these tests, where the fringe pattern's size is of 256×256, was 11.1 seconds for the RPT and 4.224 seconds for the LAQF.

As one can see in Fig. 2, the phase obtained in all cases with the LAQF method looks like the ground truth phase shown in Fig. 3, unlike the phase obtained with the RPT method, which differs from the ground truth phase as the noise grows.

In the second test, we show an experimental interferogram with closed fringes generated using a moiré technique to measure deformations on surfaces. In Fig. 4(a), we see the image interferogram, and in Fig. 4(b) its obtained phase with the LAQF. The parameters used in this test are $\lambda = 5$ and neighborhood Γ size 8×8.

The next experiment shows what happens when the image interferogram has very low frequency zones. In Fig. 5(a), we see an image interferogram with closed fringes generated with a moiré technique. This image interferogram has zones where its frequency is practically zero. In Fig. 5(b), we see the obtained phase using the LAQF. As we can see, the LAQF method behaves correctly in zones where the frequency is practically zero.

Finally In Fig. 6(a), we show another image interferogram. In this case, the fringes are available only in a zone of the image. To demodulate this kind of interferograms, a mask is used to indicate the zone where the fringes exist. As we can see, the phase is obtained correctly.

4. Discussion and conclusions

The LAQF method, developed here, is a robust demodulation method for the recovery of phase from single fringe patterns with closed fringes without the need of normalization. This method, compared to the RPT method, tolerates higher levels of noise. As the RPT method, this method is a sequential method that uses a scanning strategy following the fringes.

The method to choose the Γ neighborhood size used for the LAQF, is the same method used to choose the neighborhood size for the RPT. This is by testing different sizes. In this way, we found that neighborhood sizes of $n \times n$ for $n \in \{5, 6, 7, 8\}$ are suitable to demodulate correctly an image interferogram with closed fringes. Actually there is no rule to say what is the optimum neighborhood size to use, nither for the other sequential methods like the RPT.

The RPT method is a demodulation method that supports certain levels of noise. Other modified RPT methods that support variations in the modulation term from the fringe pattern, have been presented to demodulate single fringe patterns with closed fringes (see [13, 12, 14]). However, all these methods, as they are based on the RPT method, support the same levels of noise than the RPT. On the other hand, the LAQF approach estimates the modulating phase using local robust quadrature filters. The single common process between the RPT method and the LAQF method, presented here, is the fringe following scanning strategy used to demodulate the image interferogram.
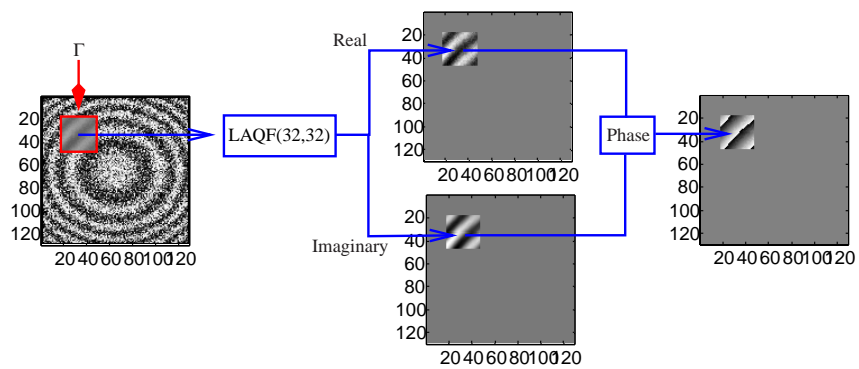
Fig. 1. In this figure, we graphically illustrate how the phase in region Γ around site $(32, 32)$ is obtained. The left image is the given interferogram where we mark the neighborhood in red. The intermediate images are the real part and imaginary part of the local quadrature filter, or LAQF, obtained after minimizing Eq. (10). Finally, the right image is the local phase obtained from the RQF.
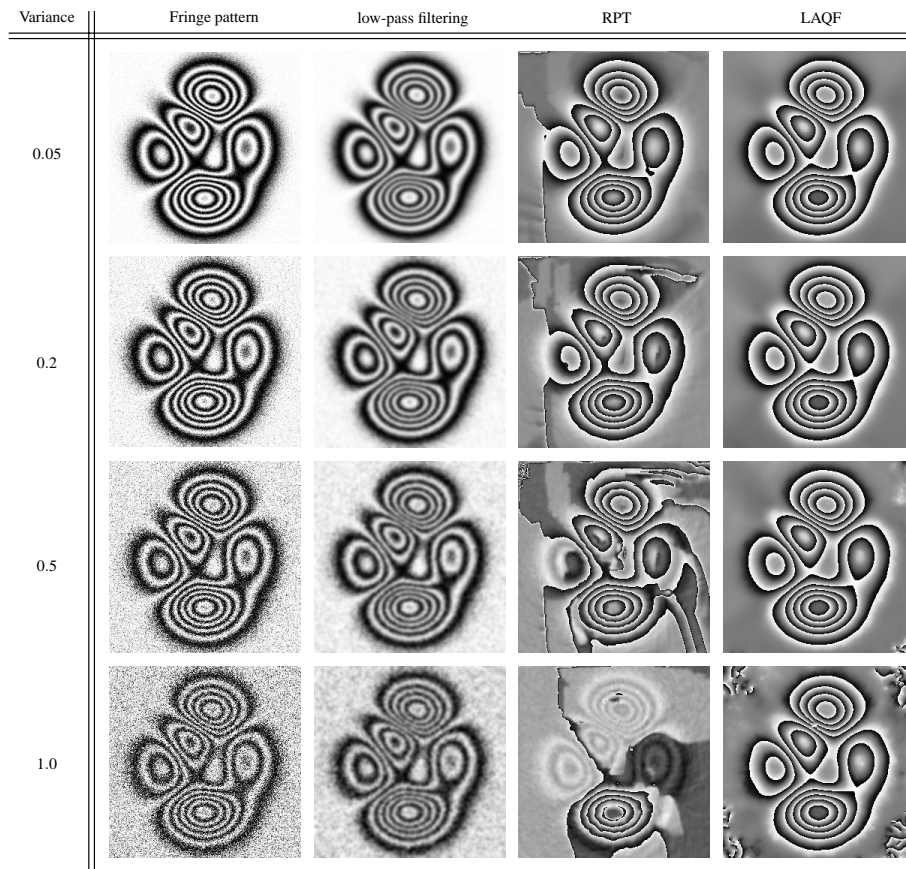
Variance	Fringe pattern	low-pass filtering	RPT	LAQF
0.05				
0.2				
0.5				
1.0				

Fig. 2. In this figure: the variance column shows the variance of the Gaussian noise added to the ground truth phase; the fringe pattern column shows the generated fringe pattern; the RPT column shows the obtained phase using the RPT method and the LAQF column the phase obtained using the LAQF.
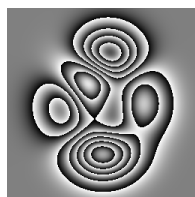

Fig. 3. Ground truth phase used to generate the fringe patterns shown in Fig. 2.
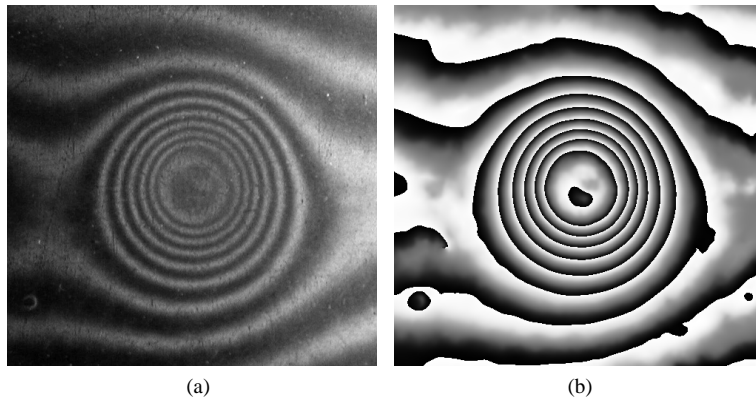
Fig. 4. (a) image interferogram with closed fringes obtained by means of a moiré technique. (b) expected obtained phase using LAQF. The image interferogram dimensions are 488×500.
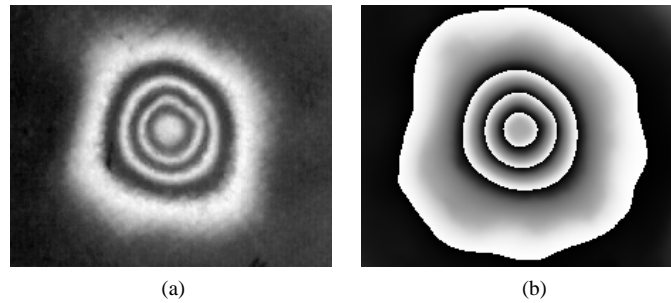

Fig. 5. (a) image interferogram with very low frequency zones. The image interferogram was generated with a moiré technique. (b) phase obtained using the LAQF.
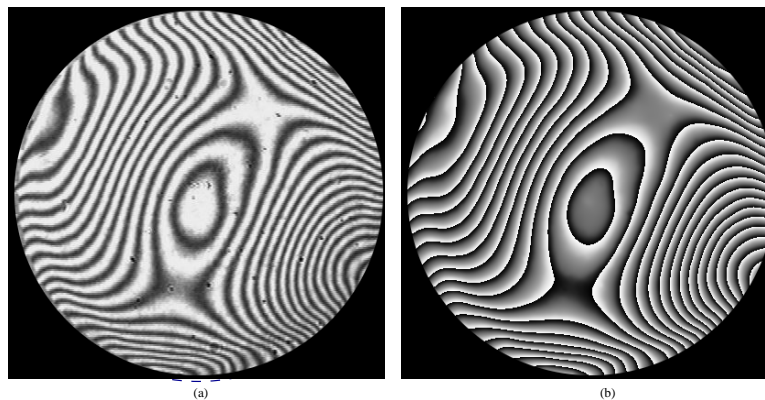

Fig. 6. (a) is the interferogram to demodulate, and (b) its demodulated phase using the LAQF. In this example, we see that the fringes exist just in a zone of the image.