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Mathematics

Ultra-Newtonian Gravitation

Vladimir Boltyanski*

* *CENTRO DE INVESTIGACIÓN EN MATEMÁTICAS A.C.*

(Presented by Academy Member G. Kharatishvili)

ABSTRACT. In the article a theory of the gravitation is given that contradicts the Einsteinian general relativity theory. The presented theory is based on the *postulate of flowing space* that was formulated by the author. The postulate means that the light velocities in direction to the central mass and in the opposite direction have the values which do not coincide. From the postulate of flowing space we deduce (very easily) Schwarzschild's metric. Furthermore, the gravitational redshift formula is proved that is postulated (without any proof) in the general relativity theory. Moreover, it is shown that a gravitational blueshift is possible. Some experiments are described two of which can discover the blueshift, whereas the third one can resolve the discussion between the Einsteinian gravitation law and the postulate of flowing space. Finally, the gravitational redshift of distant objects of the universe is explained without Hubble's hypothesis on "expanding universe". At the end of the article it is shown that the presented theory can be justified by consideration of a flux of some particles (in contrast to the general relativity theory that has, because of influence of Hilbert, purely geometrical, non-physical character). *© 2007 Bull. Georg. Natl. Acad. Sci.*

Key words: *gravitation, Einsteinian law, redshift, universe, Hubble hypothesis, light velocity.*

1. Introduction.

In 1915 Einstein published an article on the *general relativity theory.* It contained a geometrical picture of the gravitation more exact than Newtonian gravitation law.

The following visual explanation ("mental experiment" by Feynman's expression) shows the initial Einsteinian idea. If we put a heavy ball M on a strongly tight piece of a cloth, then we obtain a deepening on the cloth. A light marble placed on the cloth rolls to M. Even if we push the marble perpendicularly to the direction going to M, then it moves around M until (because of the friction and decrease of the velocity) it falls to M. Abstracting from the friction, we may say that the curvature of the cloth creates the rotation of the marble around M. At the end of XIX century physicists said on an elastic medium ("ether") filling the empty space in which the light spreads as the sound spreads in the air. Einstein supposed that the mass M bends the medium, and this explains the moving of the planets (although it is unclear how the mass can bend the surrounding medium).

Einstein turned to the well-known mathematician Hilbert with the question, how is it possible to describe mathematically the curvature of the space? Hilbert sent to Einstein several letters in which he explained what is Riemann metric of a curved space, what are Christoffel's symbols, and what is the curvature tensor. Considering the Einsteinian physical idea, Hilbert understood that in this case it is necessary to use a convoluted curvature tensor $R_{\alpha\beta}$ named the *Ricci tensor.* The vanishing of that tensor gives the mathematical description of the Einsteinian idea.

In a time Einstein published the paper on the general relativity theory, under only his name, without mentioning of Hilbert. Now *Einsteinian law of gravitation* is expressed in a fine geometrical form: *In the empty space the Ricci tensor* $R_{\alpha\beta}$ *is identically equal to zero.* Empty space means that it does not contain any substance and any field except the gravitational one.

Mathematically elegant description of the general relativity theory is given in Dirac's book [5], where at first a compact description of Riemannian theory of curvature is given and then the Einsteinian law of gravitation with some conclusions is explained. Note only that instead of indication of the names of greatest mathematicians Gauss, Riemann, Hilbert who support the Einsteinian law the author writes that "some results are applied which were obtained mainly in the last century".

The components of the Ricci tensor $R_{\alpha\beta}$ are expressed by Christoffel's symbols $\Gamma^{\alpha}_{\mu\nu}$ and their first derivatives. Furthermore, the Christoffel's symbols are expressed by the first derivatives of the components of the metric tensor $g_{\alpha\beta}$ that defines the invariant spacetime interval $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$. Here and in the sequel, as in the Riemann geometry, if the same index is twice encountered (once above and once below), then the summation over that index is made. The Greek indices take the values 0, 1, 2, 3 and the Latin indices take the values 1, 2, 3.

The equalities $R_{\alpha\beta} = 0$ form a complicated nonlinear system of differential equations of second order with partial derivatives. For a curved (non-planar) space the only solution of that system was obtained by the German physicist Schvarzschild. His solution is related to the case of a *static, spherically symmetric* gravitational field. The field is static if g_{00} does not depend on the time and, moreover, $g_{0m} = 0$ (see the deduction of the Schvarzschild's solution in section 18 of the book [5]).

The equalities $g_{0m} = 0$ enclosed in the definition of a static field mean that the value of the light velocity in two opposite directions coincide. Thus, to the Einsteinian postulate that the mass bends the enclosing space it is necessary to add the postulate which affirms that the values of the light velocities "there" and "back" coincide. Note that in all physical experiments the average velocity "there" and "back" is considered (or, as the physicists say, the light velocity "along the closed way").

Schvarzschild's metric implied some conclusions (the rotation of Mercouri's perihelion, the bend of light trajectories near the Sun) which are well confirmed by some observations. Thus, the Einsteinian gravitational law became famous.

2. The structure of the article.

In this article we outline an alternative theory named here ultra-Newtonian one. Some of its results are published in [1-4].

Consider a static Galilean system $\mathcal{G}$ that is referred to an orthonormal coordinate system x^1, x^2, x^3. Assume that at the origin of the system $\mathcal{G}$ a non-rotating, spherically symmetric mass M rests. By $t = x^0$ denote the Galilean time.

If under the influence of the Newtonian potential $\varphi = \dfrac{GM}{r}$ created by M a mass point moves from the infinity, where it had the vanishing initial velocity, then the mass point moves along the ray going to M. In the distance r from the origin it has the velocity $\bar{\upsilon}$ with $\|\bar{\upsilon}\| = \sqrt{2\varphi} = \sqrt{\dfrac{2GM}{r}}$. The corresponding acceleration is equal to $\|\bar{a}\| = \dfrac{d\|\bar{v}\|}{dr} = \dfrac{d\|\bar{v}\|}{dt} \cdot \|\bar{v}\| = \dfrac{2GM}{r^2}$ in the total conformity with the Newtonian gravitational law. Thus, if a neighborhood of the mass M is filled by mass points of negligible mass, then the whole space enclosing M as if "flows" to M, being subjected to the free fall. This is just the intuitive picture that replaces in our theory the Einsteinian intuitive picture on the mass M bending a strongly tight piece of a cloth.

Thus, instead of the Einsteinian postulate which affirms that the mass bends the enclosing space we introduce the postulate which affirms that the space enclosing the mass M as if "flows" to M with a velocity depending on the distance r from M. A small volume of the "flowing space" (a freely falling laboratory) is an inertial system in which the laws of the partial relativity theory hold. The exact statement of the "flowing space" postulate with the deduction of Schvarzschild's metric in the Galilean system are contained in section 3.

In section 4 Ricci's tensor in the system $\mathcal{G}$ is calculated. We show that $R_{00} = 0$, $R_{0p} = 0$, but $R_{pq} \neq 0$, contradicting the Einsteinian gravitation law.

In section 5 we describe the irradiation of an atom in the Galilean system $\mathcal{G}$ and the *variation of the light velocity* along the trajectory (again in the system $\mathcal{G}$). For example, if an atom rests in the surface of a star, then it is not immovable in the freely falling inertial laboratory, but is immovable in the corresponding Galilean system $\mathcal{G}$ that is not inertial. Hence the irradiation of that atom is subordinated to the laws which are distinct from ones of the relativity theory. Irradiation law in the Galilean system proved in section 5 implies, firstly, a mathematical deduction of the formula for the gravitational redshift (that is *postulated without any proof* in the general relativity theory) and, secondly, the principial possibility of the gravitational *blueshift*. We describe some experiments which allow to discover that gravitational blueshift.

Note that the experiment of Paund-Rebka (1959) and the experiment of Paund-Snider (1965) which are based on using of Mossbauer's effect discover gravitational redshift under the influence of gravitational field of the Earth. It is possible that the converted experiments (which measure gravitational shift not in the direction to the center of the Earth, but in the opposite direction) also will discover a gravitational blueshift.

In section 6 an experiment is described the result of which (if positive) can resolve the discussion between the Einsteinian gravitational law and the postulate of "flowing space" in behalf of the last one.

Section 7 contains the description of Lorentz' transformations in the Galilean system. Nevertheless, the difference from their description in an inertial system is too small for an experimental verification (in a laboratory on the Earth).

In section 8 we show that the redshift of the distant objects of the universe can be explained in the frames of the presented ultra-Newtonian gravitational theory *without* using Hubble's hypothesis on "expanding universe".

Finally, in section 9 we show that, in contrast to the general relativity theory that has (because of the influence of Hilbert) pure geometrical, non-physical character, the presented ultra-Newtonian gravitation theory can be justified with consideration of a flux of some particles.

Thus, the laws of relativity theory in the Galilean, non-inertial system form the main contents of the article.

3. Locational time.

Instead of the Einsteinian postulate which affirms that "the mass bends the space" we introduce the following

Postulate of the flowing space. *The enclosing space of the mass M "flows" to M with a velocity* $\bar{\upsilon} = \frac{d\bar{x}}{dt}$ *whose value depends on the distance of the point* $\bar{x}$ *from M and is equal to zero at the infinity. The corresponding acceleration* $\bar{a} = \frac{d\bar{\upsilon}}{dt}$ *has a scalar potential* φ*, i.e.,* $\bar{a} = \mathrm{grad}\varphi$*. The potential* φ *satisfies the Laplace equation*

$$\left(\frac{\partial^2}{\left(\partial x^1\right)^2}+\frac{\partial^2}{\left(\partial x^2\right)^2}+\frac{\partial^2}{\left(\partial x^3\right)^2}\right)\varphi=0. \tag{1}$$

Theorem 1. *The velocity of the "flowing space" has at the point* $x = (x^1, x^2, x^3)$ *the value* $\sqrt{2\varphi}$*, where* $\varphi = \frac{GM}{r}$ *is the Newtonian gravitational potential that is created by the mass M and is equal to zero at the infinity, i.e.,*

$$\bar{\upsilon} = -\sqrt{\frac{2GM}{r}}\,\bar{e}; \tag{2}$$

here r is distance of the point $\bar{x}$ *from the origin and* $\bar{e} = \frac{1}{r}\bar{x} = \left(\frac{x^1}{r}, \frac{x^2}{r}, \frac{x^3}{r}\right)$ *is the unit vector directed from M.*

Proof. Equation (1) has a unique spherically symmetric solution (up to a constant summand and a constant coefficient):

$$\varphi = \frac{GM}{r} + \text{const.}$$

Let $\upsilon(r)$ and $a(r)$ be the scalar functions with $\bar{\upsilon} = \upsilon(r)\bar{e}$ and $\bar{a} = a(r)\bar{e}$. Then $\upsilon(r) = \frac{dr}{dt}$, $a(r) = \frac{d\upsilon(r)}{dt} = \frac{d\varphi}{dr}$ and hence

$$d\left(\upsilon(r)^2\right) = 2\upsilon(r)\,d\upsilon(r) = 2\upsilon(r)\,a(r)\,dt = 2a(r)\,dr = 2d\varphi.$$

This implies

$$\upsilon(r)^2 = \frac{2GM}{r} + C\,.$$

The integration constant C is equal to zero, since the velocity $\upsilon(r)$ tends to zero when $r \to \infty$. Thus, $\upsilon(r)^2 = \frac{2GM}{r}$ and hence

$$\upsilon(r) = -\sqrt{\frac{2GM}{r}} \tag{3}$$

(the sign minus is taken, since, by the postulate of flowing space, the velocity is directed to the central mass M). ■

Make an important note. Let $\bar{x}$ be a point in the distance r from the origin. Denote by $\mathcal{G}_{\bar{x}} = \{dx^1_{\mathcal{G}}, dx^2_{\mathcal{G}}, dx^3_{\mathcal{G}}\}$ the Galilean system obtained from $\mathcal{G}$ by the translation of the origin to the point $\bar{x}$ and by $\mathcal{F}_{\bar{x}} = \{dx^1_{\mathcal{F}}, dx^2_{\mathcal{F}}, dx^3_{\mathcal{F}}\}$ the system that is fixed in the flowing space, has the origin at the point $\bar{x}$ (at the moment $t = 0$), and has the same direction of the axes as the system $\mathcal{G}_{\bar{x}}$. Using the notation $d\bar{x}_{\mathcal{G}} = \{dx^1_{\mathcal{G}}, dx^2_{\mathcal{G}}, dx^3_{\mathcal{G}}\}$ and $d\bar{x}_{\mathcal{F}} = \{dx^1_{\mathcal{F}}, dx^2_{\mathcal{F}}, dx^3_{\mathcal{F}}\}$, the passage from $\mathcal{G}_{\bar{x}}$ to $\mathcal{F}_{\bar{x}}$ is described by the formulas

$$d\bar{x}_{\mathcal{F}} = d\bar{x}_{\mathcal{G}} - \bar{\upsilon}dt_{\mathcal{G}}, \ \ dt_{\mathcal{F}} = dt_{\mathcal{G}}. \tag{4}$$

If a mass point is immovable in the flowing space, i.e., has the velocity 0 with respect to $\mathcal{F}_{\bar{x}}$, then its velocity with respect to the Galilean system $\mathcal{G}_{\bar{x}}$ is equal to $\bar{\upsilon}$. Even if a mass point has a velocity $\bar{u}$ with respect to $\mathcal{F}_{\bar{x}}$, then its velocity in the Galilean system $\mathcal{G}_{\bar{x}}$ is equal to $\bar{u} + \bar{\upsilon}$.

Since the local system $\mathcal{F}_{\bar{x}}$ is inertial and the passage (4) is distinct from the Lorentz transformations, the Galilean system $\mathcal{G}_{\bar{x}}$ *is not inertial*. Consequently the value of the light velocity in the Galilean system is distinct from c.

A photon has (in the flowing space) its own velocity $\bar{c}$ of the value c. With respect to the Galilean system the photon has the velocity $\bar{c} + \bar{\upsilon}$, where $\bar{\upsilon}$ is described by formula (2). In particular, the photon going to M has the superlight velocity $c_1 = c + \upsilon$ and the photon going from M has the sublight velocity $c_2 = c - \upsilon$.

In the Galilean system $\mathcal{G}_{\bar{x}}$ all photons passing at a moment through the point $\bar{x}$ will be in a time $dt_{\mathcal{G}}$ situated in the surface of the sphere of radius $cdt_{\mathcal{G}}$ centered at the point $\bar{x} + \bar{\upsilon}dt_{\mathcal{G}}$ (Fig. 1). That sphere consists of all points $\bar{x} + d\bar{x}_{\mathcal{G}}$ which are situated in the distance $cdt_{\mathcal{G}}$ from the point $\bar{x} + \bar{\upsilon}dt_{\mathcal{G}}$, i.e., satisfy the condition $\|d\bar{x}_{\mathcal{G}} - \bar{\upsilon}dt_{\mathcal{G}}\| = cdt_{\mathcal{G}}$. In other words, for a given $dt_{\mathcal{G}}$ that sphere has in the Galilean system $\mathcal{G}_{\bar{x}}$ the equation $ds^2_{\mathcal{G}} = 0$, where $ds^2_{\mathcal{G}} = c^2(dt_{\mathcal{G}})^2 - \|d\bar{x}_{\mathcal{G}} - \bar{\upsilon}dt_{\mathcal{G}}\|^2$, i.e.,

$$ds^2_{\mathcal{G}} = (c^2 - \|\bar{\upsilon}\|^2)(dt_{\mathcal{G}})^2 + 2\langle\bar{\upsilon}, d\bar{x}_{\mathcal{G}}\rangle dt_{\mathcal{G}} - \|d\bar{x}_{\mathcal{G}}\|^2. \tag{5}$$

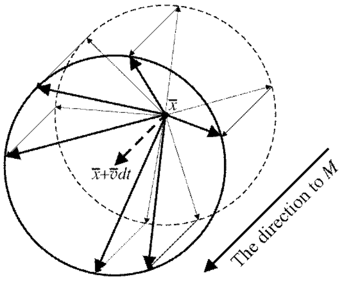

Fig. 1.

We call (5) the *main quadratic polynomial*. In the space of variables dx^0, dx^1, dx^2, dx^3 it defines a pseudo-Euclidean metric that is analogous to the pseudo-Euclidean metric of Minkowski. By virtue of (4), the main quadratic polynomial (5) can be rewritten in the system $\mathcal{F}_{\bar{x}}$ as $c^2(dt_{\mathcal{F}})^2 - \|dx_{\mathcal{F}}\|^2$, i.e., the main quadratic polynomial (5) represents the spacetime interval that is written however not in the inertial system $\mathcal{F}_{\bar{x}}$, but in the Galilean system $\mathcal{G}_{\bar{x}}$.

In the sequel, up to section 7, we will conduct reasoning only in the Galilean system $\mathcal{G}_{\bar{x}}$. Therefore we will omit (up to section 7) the index $\mathcal{G}$, i.e., we will use in $\mathcal{G}_{\bar{x}}$ the notation $d\bar{x} = (dx^1, dx^2, dx^3)$ instead of $d\bar{x}_{\mathcal{G}} = \{dx^1_{\mathcal{G}}, dx^2_{\mathcal{G}}, dx^3_{\mathcal{G}}\}$.

Let now $dt = dt_{(+)}$ be the Galilean time of spreading of a light signal from the point $\bar{x}$ to $\bar{x} + d\bar{x}$, and $dt_{(-)}$ be the Galilean time of spreading of a light signal from $\bar{x} + d\bar{x}$ back to $\bar{x}$. Then $dt_* = \frac{1}{2}\left(dt_{(+)} + dt_{(-)}\right)$ is the *locational* time of spreading of a light signal between $\bar{x}$ and $\bar{x} + d\bar{x}$ (the light signal emanates from $\bar{x}$ and then, "reflecting" at the point $\bar{x} + d\bar{x}$, returns to $\bar{x}$, Fig. 2). Dividing the distance $\| d\bar{x} \|$ between the points $\bar{x}$ and $\bar{x} + d\bar{x}$ by the locational time t_* we obtain the average light velocity "there" and "back", i.e., the velocity "along the closed way".

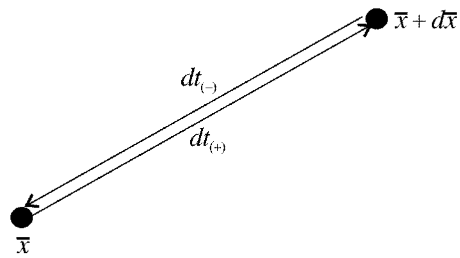

Fig. 2.

We observe the events in the locational time. Indeed, if we have only *one* clock (at the point $\bar{x}$), then we can find only the average time "there" - "back". Even if we wish to measure the light velocity *in one direction,* then it is necessary to use *two* clocks (at the points $\bar{x}$ and $\bar{x} + d\bar{x}$). In this case the clocks have to be *synchronized.* But to synchronize the clocks we need the light velocity in one direction. We obtain a vicious circle.

Theorem 2. *Under the passage from the Galilean time dt to the locational time dt_* the metric ds^2 described by the main quadratic polynomial turns to Schwarzschild's metric.*

Proof. By (5), the Galilean time $dt = dt_{(+)}$ of spreading of the signal from $\bar{x}$ to $\bar{x} + d\bar{x}$ is the *positive* solution of the equation $ds^2 = 0$. Furthermore, $dt_{(-)}$ is the positive solution of the equation obtained from the equation $ds^2 = 0$ by the substitution $-d\bar{x}$ instead of $d\bar{x}$. In other words, $-dt_{(-)}$ is the *negative* solution of the equation $ds^2 = 0$. Thus,

$$dt - dt_* = dt_{(+)} - dt_* = dt_{(+)} - \frac{dt_{(+)} + dt_{(-)}}{2} = \frac{dt_{(+)} + (-dt_{(-)})}{2}.$$

Applying Viet's Theorem on the solutions of a quadratic equation and using the expression (2) for the velocity *, we obtain from the equation $ds^2 = 0$ the equality

$$dt - dt_* = -\frac{\langle \bar{\upsilon}, d\bar{x} \rangle}{c^2 - \| \bar{\upsilon} \|^2},$$

and hence

$$dt_* = dt + \frac{\langle \bar{\upsilon}, d\bar{x} \rangle}{c^2 - \| \bar{\upsilon} \|^2}.$$

Now, extracting the perfect square in the right hand side of (5), we rewrite the main quadratic polynomial as

$$ds^2 = \left(c^2 - \| \bar{\upsilon} \|^2\right)\left(dt + \frac{\langle \bar{\upsilon}, d\bar{x} \rangle}{c^2 - \| \bar{\upsilon} \|^2}\right)^2 - \frac{\left(\langle \bar{\upsilon}, d\bar{x} \rangle\right)^2}{c^2 - \| \bar{\upsilon} \|^2} - \| d\bar{x} \|^2.$$

i.e.,

$$d\hat{s}^2 = \left(c^2 - \| \bar{\upsilon} \|^2\right) dt_*^2 - \frac{\left(\langle \bar{\upsilon}, d\bar{x} \rangle\right)^2}{c^2 - \| \bar{\upsilon} \|^2} - \| d\bar{x} \|^2.$$

Using (2), we obtain

$$ds^2 = \left(c^2 - \frac{2GM}{r}\right) dt_*\, dt_* - \left(\| d\bar{x} \|^2 + \frac{\frac{2GM}{r^3}}{c^2 - \frac{2GM}{r}} \langle \bar{x}, d\bar{x} \rangle^2 \right),$$

i.e., in coordinate form,

$$ds^2 = \left(c^2 - \frac{2GM}{r}\right) dt_*^{\,0}\, dt_*^{\,0} + \left(h_{pq} - \frac{\frac{2GM}{r^3} x_p x_q}{c^2 - \frac{2GM}{r}} \right) dx^p dx^q ,$$

where $h_{pq} = -1$ for $p = q$ and $h_{pq} = 0$ for $p \neq q$. But this is just Schwarzschild's metric (see section 18 in [5], where Schwarzschild's metric is written in spherical polar coordinates). ■

4. Ricci's tensor in Galilean system

Certainly, for Schwarzshild's metric Ricci's tensor $R_{\alpha\beta}$ vanishes (that is clear, since Schwarzshild's metric is just obtained by the resolution of the system $R_{\alpha\beta} = 0$ for $\alpha, \beta = 0,1,2,3$). Thus, it seems that the space has the "Einsteinian curvature". Nevertheless, that curvature arises *artificially,* because of the Einsteinian principle on coincidence of the light velocity values "there" and "back" (see section 18 in [4], where for deduction of Schwarzshild's metric the equalities $g_{0m} = 0$ are used), i.e., because of the passage to the locational time. The following lemma shows that for metric (5), in contrast to Schwarzshild's one, some components of Ricci's tensor are distinct from zero (in this connection, see section 6 below, where the difference between the both metrics is considered).

Lemma 1. *Let $\mathcal{G}$ be the Galilean coordinate system and at its origin a non-rotating, spherically symmetric mass M of radius r_0 rests. Then the components of Ricci's tensor for the metric defined by the main quadratic polynomial* (5) *have in the system $\mathcal{G}$ the following values (for the surrounding space of the mass M):*

$$R_{oo} = 0,\ R_{op} = R_{po} = 0,\ R_{pq} = \frac{10(GM)^2}{c^4 r^6}(x_p x_q + h_{pq}).$$

Proof. The metric (5) can be rewritten (for $r \geq r_0$) in the form

$$ds^2 = \left(c^2 - \frac{2GM}{r}\right)(dx^0\, dx^0) + 2\sqrt{\frac{2GM}{r^3}}\, h_{pq}\, dx^q\,(dx^p\, dx^0) + h_{pq}\,(dx^p\, dx^q),$$

i.e., in the form $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$, where

$$g_{00} = c^2 - \frac{2GM}{r},\quad g_{0p} = g_{p0} = \sqrt{\frac{2GM}{r^3}}\, h_{pq} x^q,\quad g_{pq} = h_{pq}.$$

Now, using the formula $\Gamma_{\mu\nu\sigma} = \frac{1}{2}\left(g_{\mu\nu,\sigma} + g_{\mu\sigma,\nu} - g_{\sigma\nu,\mu}\right)$ for Christoffel's symbol of the first kind and the standard formula $r_{,p} = -\frac{x_p}{r}$, we obtain

$$\Gamma_{000} = 0,\ \Gamma_{00p} = \Gamma_{0p0} = 0,\ \Gamma_{p0q} = \Gamma_{pq0} = \frac{1}{2}(g_{p0,q} - g_{0q,p}) = 0,\ \Gamma_{pqk} = 0,$$

$$\Gamma_{0pq} = \frac{1}{2}(g_{0p,q} + g_{0q,p}) = -\sqrt{\frac{9GM}{2r^7}}\, x_p x_q + \sqrt{\frac{2GM}{r^3}}\, h_{pq}.$$

Furthermore, we use the matrix $(g^{\alpha\beta})$ reciprocal to the matrix $(g_{\alpha\beta})$:

$$g^{00} = \frac{1}{c^2},\quad g^{0p} = g^{p0} = \frac{1}{c^2} - \sqrt{\frac{3GM}{r^3}}\, x^p,\quad g^{pq} = -h^{pq} - \frac{2GM}{r^3} x^p x^q.$$

where $h^{\alpha\beta} = 0$ for $\alpha \neq \beta$ and $h^{\alpha\alpha} = -1$. (We note that instead of the computing of the components $g^{\alpha\beta}$ it is sufficient to check that $g_{\alpha\mu} g^{\mu\beta}$ coincides with Kronecker's symbol δ^α_β, i.e., $\delta^\alpha_\beta = 1$ for $\alpha = \beta$ and $\delta^\alpha_\beta = 0$ otherwise.)

With the help of the matrix ($g^{\alpha\beta}$), it is possible to determine Christoffel's symbols of the second kind $\Gamma^{\sigma}_{\nu\mu} = g^{\sigma\alpha}\Gamma_{\alpha\nu\mu}$. We obtain

$$\Gamma^0_{00}=0,\ \Gamma^p_{00}=0,\ \Gamma^0_{p0}=0,\ \Gamma^0_{0p}=0,\ \Gamma^q_{p0}=0,\ \Gamma^q_{0p}=0$$

$$\Gamma^0_{pk}=\frac{1}{c^2}\Gamma_{0pk},\ \Gamma^q_{pk}=\frac{1}{c^2}\sqrt{\frac{2GM}{r^3}}x^q\,\Gamma_{0pk}.$$

Finally, using the formula

$$R_{\alpha\beta}=\Gamma^{\lambda}_{\alpha\lambda,\beta}-\Gamma^{\lambda}_{\alpha\beta,\lambda}+\Gamma^{\rho}_{\alpha\lambda}\Gamma^{\lambda}_{\rho\beta}-\Gamma^{\rho}_{\alpha\beta}\Gamma^{\lambda}_{\rho\lambda},$$

we obtain (with the help of some identical, but enough complicated calculation) the components of Ricci's tensor $R_{\alpha\beta}$ indicated in the lemma. ■

Thus, metric (5) does not satisfy the Einsteinian law of gravitation. Nevertheless, it satisfies the Einsteinian law approximately, up to infinitesimal of the fourth order with respect to $\frac{\|\upsilon\|}{c}$. Indeed, since the inequality $\frac{|x|}{r}<1$ holds and, by (3), $\left(\frac{\|\upsilon(r)\|}{c}\right)^4=\left(\frac{2GM}{rc_2}\right)^2$ we obtain

$$R_{pq}=\frac{5}{2r^2}\cdot\left(\frac{2GM}{rc_2}\right)^2\cdot\left(\frac{x_p}{r}\frac{x_q}{r}+\frac{h_{pq}}{r^2}\right)=O\left(\frac{\|\upsilon(r)\|}{c}\right)^4.$$

5. Gravitational redshift and blueshift.

Consider the radiation of an atom resting at a point $\bar{x}_0$ in the surface of a star that has mass M and radius r_0. In the Einsteinian general relativity theory it is postulated that at the radiation moment the *gravitational redshift* arises, i.e., an increase of the wave length:

$$\frac{\Delta\lambda}{\lambda}\approx\frac{GM}{r_0c^2},$$

but *during the spreading along the light trajectory the wave length does not change.* We name the affirmation written with italic by Dirac's postulate (see section 17 in [5]). In particular, for the Sun $\frac{\Delta\lambda}{\lambda}\approx 2.1\cdot10^{-6}$; experiments confirm this value.

We give another model. There is no gravitational redshift at the radiation moment, but along the light trajectory in the Galilean system the wave length *does not conserve a constant value.* This is possible, since in the Galilean system the light velocity value is not constant.

The following theorem gives the law of changing of the wave length along the light trajectory in the Galilean system. In its statement (and in the sequel) the formulas are written up to infinitesimal of higher order with respect to $\frac{\upsilon^2}{c^2}$.

Theorem 3. *Assume that a photon is radiated by an atom that is immovable in the Galilean system (for example, by an atom resting in the surface of a star that has mass M and radius r_0). Furthermore, assume that at the radiation moment the photon has the wave length λ_0 and it has the wave length λ at a point $\bar{x}$ in distance r from the origin, i.e., from the center of M (the wave length is considered in the Galilean system). Then*

$$\frac{\Delta\lambda}{\lambda_0} = -\frac{\Delta\varphi}{c^2} = \frac{1}{2}(\varphi(\bar{x}_0) - \varphi(\bar{x})),$$

where $\Delta\lambda = \lambda - \lambda_0$ *and* $\varphi = \frac{GM}{r}$ *is the Newtonian gravitational potential.*

Proof. If an atom rests in the surface of a star that has mass M and radius r_0, then it is immovable in the corresponding Galilean system.

The mass of the photon is equal to $m = \frac{\hbar\nu}{c^2}$, where ν is its frequency. The work required for the transference of the photon from the point $\bar{x}$ situated in the distance r to a point $\bar{x} + d\bar{x}$ in the distance $r + dr$ from the origin is equal to $-\frac{\hbar\nu}{c^2}d\varphi$. If $dr > 0$, then the potential energy of the photon increases and hence its kinetic energy $\hbar\nu$ decreases. Therefore $\hbar d\nu = \frac{\hbar\nu}{c^2}d\varphi$, i.e., $\frac{d\nu}{\nu} = \frac{d\varphi}{c^2}$. Integrating, we obtain $\Delta \ln\nu = \frac{\Delta\varphi}{c^2}$. If the gravitation is not large, then $\frac{\Delta\nu}{\nu}$ is small, and we have $\Delta \ln\nu = \ln\frac{\nu + \Delta\nu}{\nu} = \frac{\Delta\nu}{\nu}$. Thus, $\frac{\Delta\nu}{\nu} = \frac{\Delta\varphi}{c^2}$. Furthermore, since $\lambda d\nu = -\nu d\lambda$, we conclude that $\frac{\Delta\lambda}{\lambda} = \frac{\Delta\varphi}{c^2}$. ■

If a photon recedes, then $\Delta\lambda > 0$, since $r > r_0$, i.e., we obtain a gravitational redshift (Fig. 3). In particular, if $r \to \infty$, Theorem 3 implies $\frac{\Delta\lambda}{\lambda_0} = \frac{GM}{r_0 c^2}$, i.e., we obtain the formula that is postulated in the general relativity theory.

If a photon moves from a point $\bar{x}_0$, where it had the wave length λ_0, to a mass M with an observer in its surface, then by Theorem 3 the wave length perceptible by the observer is *lesser* than λ_0 (Fig 4), i.e., by Theorem 3, a gravitational *blueshift* appears. We describe two experiments which can confirm that blueshift.

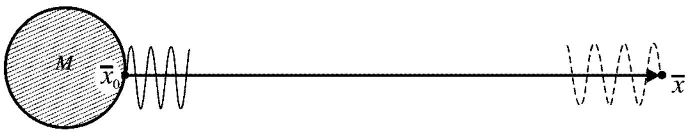

Fig. 3.

Experiment 1. Assume that at a point $\bar{x}_0$ in distance $r_0 \approx 30\,000$km from the center of the Earth (say, on a satellite, Fig. 5) there is a laser that sends a light signal of the wave length λ_0 to the Earth. Denoting the mass and the radius of the Earth by M and r, respectively, and the wave length perceptible on the surface of the Earth by λ, we conclude from Theorem 3 that the signal is perceptible on the Earth with the *gravitational blueshift*

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{1}{c^2}\left(\frac{GM}{r_0} - \frac{GM}{r_0}\right) \approx -5.5 \cdot 10^{-10}.$$

To exclude the gravitation action of the Sun, the experiment has to be made at the sunset time, when the trajectory going from the laser is perpendicular to the direction to the Sun (Fig. 5).

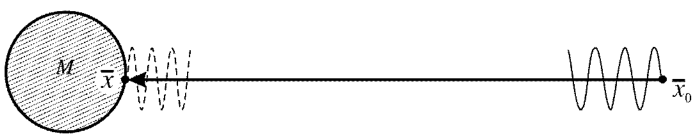

Fig. 4.

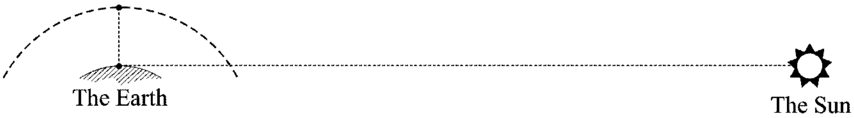

Fig. 5.

Experiment 2. Consider a rocket in the half distance between the Earth and the Sun (Fig. 6). Denote by M the mass of the Sun and by r_0 the distance from the Earth to the Sun.

Hence the rocket is situated in the distance $r = \frac{1}{2}r_0$ from the Sun. Theorem 3 implies that the light signal sent from the Earth will be perceptible on the rocket with the *gravitational blueshift*

$$\frac{\Delta\lambda}{\lambda_0} = \frac{1}{c^2}(\varphi(\bar{x}_0)\varphi(\bar{x}_0)) = \frac{1}{c^2}\left(\frac{GM}{r_0} - \frac{GM}{r_0}\right) \approx -9.9 \cdot 10^{-7}.$$

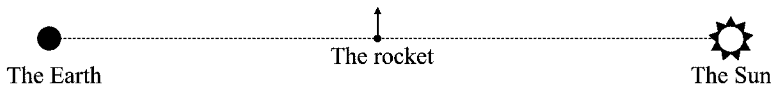

Fig. 6.

6. Comparison of the postulates.

Consider a conclusion from the postulate of flowing space directly obtained with the help of the main quadratic polynomial (5) *without* the passage to the locational time, i.e., *without* the passage to Schwarzschild's metric.

Let a spherically symmetric mass M be situated at the origin of the Galilean system. We fix a basic distance L and consider (in a laboratory in a distance r from M) two segments be and $be1$ of the length $\frac{L}{2}$ the first of which is directed to M and the second one is perpendicular to it (Fig. 7). At the points e and e_1 there are the mirrors which reflect the light going from b back to the point b. At the point b there is a semitransparent mirror. The glass plate p is included to make the length "there" and "back" the same.

The light going to M has the velocity $c_1 = c + \upsilon$, whereas the light going from M has the velocity $c_2 = c - \upsilon$. Therefore the light signal which is emanated from b, then is reflected at e and returns to b, uses for that way the time

$$\Delta t_M = \frac{L}{2(c+\upsilon)} + \frac{L}{2(c-\upsilon)} = \frac{L}{c}\left(1 + \frac{\upsilon^2}{c^2}\right).$$

Furthermore, by (5), the light velocity in the perpendicular direction (along the segments be_1 and e_1b) has the value $c_\perp = \sqrt{c^2 - \upsilon^2} = c\left(1 - \frac{\upsilon^2}{2c^2}\right)$. Therefore, the light signal which is emanated from b, then is reflected at e_1 and returns to b, uses for that way the time

$$\Delta t_\perp = \frac{L}{c_\perp} = \frac{L}{c}\left(1 + \frac{\upsilon^2}{2c^2}\right).$$

Thus, if the signals are emanated from b simultaneously, then the second signal returns to the point b *earlier* than the first one, and the lag time is equal to

$$\Delta t = \Delta t_M - \Delta t_\perp = \frac{L}{2c} \cdot \frac{\upsilon^2}{c^2}.$$

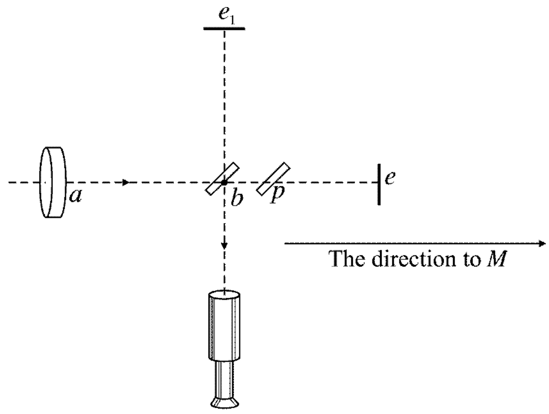

Fig. 7.

By Theorem 1 we obtain $\frac{\upsilon^2}{2c^2} = \frac{2GM}{rc^2}$, and therefore

$$\Delta t = \Delta t_M - \Delta t_\perp = \frac{L}{c} \cdot \frac{GM}{rc^2}.$$

In particular, assume that the Sun is situated at the origin of the Galilean system and the laboratory is situated on the Earth (we note that the Earth with a great exactness may be considered as a Galilean system, since its distance from the Sun is approximately constant). Since the period T of the spectral line of sodium is equal to $1.965 \cdot 10^{-15}$sec, we obtain from (5) that the equality $\Delta t = \frac{1}{2}T$ holds when $\Delta x = L = 29.7$ m. Thus for that basic distance L the lag time Δt is equal to the *semiperiod* of the spectral line of the sodium.This allows to make the following experiment.

Experiment 3. In Michelson-Morley's experiment (see. Exercise 33 to Chapter I in [6]) instead the segments be and be_1 the reflection of the light at several mirrors is considered in such a manner that the light passed a way of the length $L = 22$m. Let us repeat that experiment, taking $L = 29.7$ m and directing the interferometer to the Sun (since the interferometer has to be situated in a horizontal plane, it is necessary to make the experiment at the sunset time, Fig. 8). Then one of two light signals will have the lag time that is equal to semiperiod of the spectral line of sodium, and the field of vision in the eyepiece of the telescope will be *dark*. Even if the interferometer is rotated in 45° (such that the bisector of the angle between be and be_1 will be directed to the Sun, Fig. 9), then $\Delta t = 0$, and the field of vision in the eyepiece of the telescope will be *light*.

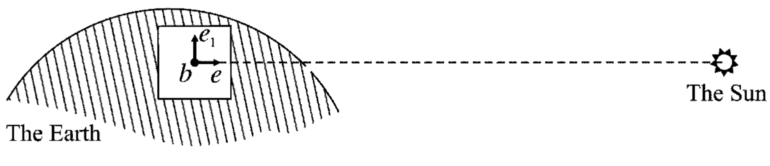

Fig. 8.

This shows the difference between the values of the light velocity (average, i.e., along a closed way) in the direction to the Sun and in the perpendicular direction.

The modern technology allows to realize experiment 3 in a simpler manner. Instead of the segment be take a spool with light-guide (and the same spool instead of the segment be_1). To exclude the gravitational influence of the Earth both the spools should be situated in such a manner that each loop of the light-guide will be situated in the horizontal plane. Now the length of light trajectory in Experiment 3 should be multiplied by $\frac{2}{\pi}$, since $\frac{1}{2\pi}\int_0^{2\pi} |\cos\alpha|\, d\alpha = \frac{2}{\pi}$. Thus, the length of the light-guide (in each one of the spools) should be replaced by $L \cdot \frac{\pi}{2} = 29.7 \cdot 1.57$ m$=46.6$ m (certainly, if we use the same spectral line of sodium).

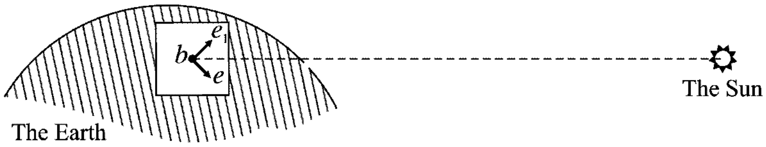

Fig. 9.

We note that, by Lemma 1, metric (5) does not satisfy the Einsteinian law of gravitation. Moreover, it does not satisfy the Einsteinian postulate on coincidence of the values of light velocity "there" and "back", i.e., metric (5) does not satisfy the main postulates of the general relativity theory. The positive result of Experiment 3 will mean that the postulate of flowing space describes the real processes more exactly than Schwarzschild's metric (that satisfies the main postulates of the Einsteinian general relativity theory). Thus, the controversy between the Einsteinian general relativity theory and the presented here ultra-Newton theory will be solved in behalf of the last one.

Furthermore, Laplace's equation is *linear* and hence, summing the potentials of the type $\frac{GM}{r}$ for arbitrary distribution of masses, we again obtain a solution of Laplace's equation. At the same time, Ricci's tensor is *non-lineal.* It is possible that a double star gives another counterexample to the Einsteinian gravitation law.

7. Lorentz' transformations in Galilean system

Let $\mathcal{G}$ be a Galilean coordinate system and at its origin a non-rotating, spherically symmetric mass M rests. We consider the orthonormal coordinate systems $\mathcal{G}_{\bar{x}} = \left\{dx^1_{\mathcal{G}}, dx^2_{\mathcal{G}}, dx^3_{\mathcal{G}}\right\}$ and $\mathcal{F}_{\bar{x}} = \left\{dx^1_{\mathcal{F}}, dx^2_{\mathcal{F}}, dx^3_{\mathcal{F}}\right\}$ introduced after the proof of Theorem 1.

Let now $\mathcal{F}'_{\bar{x}} = \left\{dx'^1_{\mathcal{F}}, dx'^2_{\mathcal{F}}, dx'^3_{\mathcal{F}}\right\}$ be the system that moves uniformly with respect to the flowing system $\mathcal{F}_{\bar{x}}$ with the velocity $\bar{u}$ that is directed to the origin and has the value $\|\bar{u}\| = u$. We assume that the system $\mathcal{F}'_{\bar{x}}$ has the same directions of axes as $\mathcal{F}_{\bar{x}}$ and at the moment $t = 0$ both the systems have the coinciding origins. Moreover, assume that the axis x^1 is directed to the origin and the axes x^2, x^3 have perpendicular directions.

Since the systems $\mathcal{F}_{\bar{x}}$ and $\mathcal{F}'_{\bar{x}}$ are inertial, the passage from $\mathcal{F}_{\bar{x}}$ to $\mathcal{F}'_{\bar{x}}$ is described by Lorentz' transformations:

$$dx^1_{\mathcal{F}'} = \frac{1}{W}(dx^1_{\mathcal{F}} - u\,dt_{\mathcal{F}}), \quad dt_{\mathcal{F}'} = \frac{1}{W}\left(-\frac{u}{c^2}dx^1_{\mathcal{F}} + dt_{\mathcal{F}}\right), \tag{7}$$

where $W = \sqrt{1 - \frac{u^2}{c^2}}$.

Now we pass to the Galilean system $\mathcal{G}_{\bar{x}}$. Denote by $(dx'^1_{\mathcal{G}}, dt'_{\mathcal{G}})$ coordinates of the point $(dx^1_{\mathcal{F}'}, dt_{\mathcal{F}'})$ and by $(dx^1_{\mathcal{G}}, dt_{\mathcal{G}})$ coordinates of the point $(dx^1_{\mathcal{F}}, dt_{\mathcal{F}})$ in the Galilean system $\mathcal{G}_{\bar{x}}$. Applying (4) to Lorentz' transformations (7), we obtain

$$dx'^1_{\mathcal{G}} - \upsilon dt'_{\mathcal{G}} = \frac{1}{W}((dx^1_{\mathcal{G}} - \upsilon dt_{\mathcal{G}}) - u\,dt_{\mathcal{G}}), \quad dt'_{\mathcal{G}} = \frac{1}{W}\left(-\frac{u}{c^2}(dx^1_{\mathcal{G}} - \upsilon dt_{\mathcal{G}}) + dt_{\mathcal{G}}\right).$$

Making identical algebraic transformations, we obtain the following lemma.

Lemma 2. *Let $\mathcal{F}'_{\bar{x}}$ be the system that moves uniformly with the velocity $\bar{u}$ with respect to the system $\mathcal{F}_{\bar{x}}$. If $\bar{u}$ is parallel to $\bar{\upsilon}$, then the corresponding Lorentz' transformations have in the Galilean systems $\mathcal{G}_{\bar{x}}$ the following form:*

$$dx'^1_{\mathcal{G}} = \frac{1}{W}\left(\left(1 - \frac{u\upsilon}{c^2}\right)dx^1_{\mathcal{G}} - u\left(1 - \frac{\upsilon^2}{c^2}\right)dt_{\mathcal{G}}\right),$$

$$dt'_G = \frac{1}{W}\left(-\frac{u}{c^2}dx^1_G + \left(1+\frac{u\upsilon}{c^2}\right)dt_G\right).$$

Even if the vector $\bar{u}$ *is perpendicular to* $\bar{\upsilon}$ *, say, is directed along the axis* x^2*, then Lorentz' transformations have the form analogous to* (7):

$$dx'^2_G = \frac{1}{W}(dx^2_G - u\,dt_G)\,, \qquad dt'_G = \frac{1}{W}\left(-\frac{u}{c^2}dx^2_G + dt_G\right). \;\blacksquare$$

The above lemma means that in the Galilean system the main quadratic polynomial is invariant under Lorentz' transformations indicated in the lemma (this affirmation can be verified directly, too). Moreover, Lemma 2 implies that if $\bar{u}$ is vertical or horizontal, then, respectively,

$$\frac{dx^1_G}{dt_G} = u\cdot\frac{1-\frac{\upsilon^2}{c^2}}{1-\frac{u\upsilon}{c^2}} \approx u\cdot\left(1-\frac{\upsilon^2}{c^2}+\frac{u\upsilon}{c^2}\right), \qquad \frac{dx^2_G}{dt_G} = u\,.$$

This is an essential difference between the ultra-Newtonian gravitation theory and the Einsteinian one. Nevertheless, that difference is too small for an experimental verification.

8. Redshift of distant objects of the universe

The observation shows that the radiation of the distant objects of the universe has the gravitational redshift proportional to their removal. The accepted cause is Hubble's hypothesis on the expansion of the universe, as the result of which Doppler's effect of distant objects arises. In contrast to that accepted point of view, in the presented ultra-Newtonian gravitation theory the following affirmation holds:

Theorem 4. *In the frames of the ultra-Newtonian gravitation theory it is possible to construct without Hubble's hypothesis a model of the universe in which the gravitational redshift is proportional to the distance of the objects.*

A sketch of a proof is given in [4]. For completeness of the article we give here a modified proof.

Proof. Imagine the "universe" as a ball of radius R^* centered at the origin of the Galilean system. Assume that the ball B_r of radius r contains the spherically symmetric mass $M(r) = \frac{2}{\pi}M^* \arctan\left(\frac{\pi}{2}\cdot\frac{kr^2}{R^*-r}\right)$, where $M^* = \frac{c^2 R^*}{G}$.

Then the density of the masses tends to zero when $r \to R$.

Let $\bar{x}$ be a point in distance r from the origin. According to the potential theory, the gravitational action of the exterior part of the ball B_r is equal to zero at the point $\bar{x}$. Hence at $\bar{x}$ the potential being made by the whole "universe" *coincides* with the gravitational action of the ball B_r, i.e., $\varphi(\bar{x}) = \frac{GM(r)}{r}$. At the origin $\varphi = 0$. By Theorem 3 , for the light going from $\bar{x}$ to the origin we have $\frac{\Delta\lambda}{\lambda} \approx kr$ if r is not very great; indeed,

$$\arctan\left(\frac{\pi}{2}\cdot\frac{kr^2}{R^*-r}\right) \approx \frac{\pi}{2}\cdot\frac{kr^2}{R^*-r} \approx \frac{\pi}{2}\cdot\frac{kr^2}{R^*}$$

and therefore

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\varphi(\bar{x})}{c^2} = \frac{GM(r)}{rc^2} \approx \frac{G}{rc^2}\cdot M^*\cdot\frac{kr^2}{R^*} = kr.$$

Moreover, $\frac{\Delta\lambda}{\lambda} < 1$ for all $r < R^*$. For the points distinct from the origin the picture is analogous.

We note that in the described model of the “universe” the gravitational shift becomes to be *blue* when the point $\bar{x}$ approaches to the boundary of the “universe”, i.e., when $r \to R^*$. ■

9. Flux of tachyons

Naturally the question arrises how the postulate of the “flowing space” can be justified? We suggest the following hypothetic explanation. The mass M emanates a spherically symmetric flux of which have neither mass nor energy, but the value of the velocity of tachyons is essentially greater than the light velocity c (in the flowing space).

Interacting with a mass point (or with a photon), the tachyon as if “drags up” it, i.e., the mass point (or a photon) is a little displaced to the mass M. The displacement obtained because of the “tachyon wind” causes a velocity of a mass point (or a photon) in direction to M (Fig. 10), i.e., the space as if “flows” in direction to the mass M. There is a more detailed explanation of the action of the tachyons in articles [2, 3].

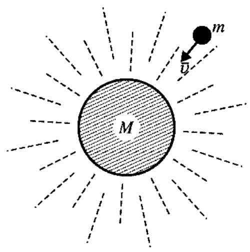

Fig. 10.

Thus, in contrast to the General Relativity Theory, it is possible to explain the ultra-Newtonian gravitation with the help of a flux of some particles which are named here *tachyons*. And if the photons create the time, then the tachyons create the flowing space.

მათემატიკური ფიზიკა

ნიუტონის ზეგრავიტაცია

ვ. ბოლტიანსკი*

* *მათემატიკის კვლევითი ცენტრი*

(წარმოდგენილია აკადემიკოს გ. ხარატიშვილის მიერ)

ნაშრომში მოცემულია გრავიტაციის თეორია, რომელიც ეწინააღმდეგება აინშტაინის ფარდობითობის ზოგად თეორიას. წარმოდგენილი თეორია დაფუძნებულია ავტორის მიერ ჩამოყალიბებულ დინებადი სივრცის პოსტულატზე. ეს პოსტულატი გულისხმობს, რომ სინათლის სიჩქარეს ცენტრის მიმართულებით და მის საწინააღმდეგოდ ერთმანეთისგან განსხვავებული მნიშვნელობები აქვს. დინებადი სივრცის პრინციპიდან გამომდინარე მიღებულია შვარცშილდის მეტრიკა. გარდა ამისა, გამოყვანილია გრავიტაციული წითელი წანაცვლების ფორმულა, რომელიც ფარდობითობის ზოგად თეორიაში პოსტულირებულია დამტკიცების გარეშე. ნაჩვენებია აგრეთვე, რომ შესაძლებელია იისფერი წანაცვლება. აღწერილია რამდენიმე ექსპერიმენტი, რომელთაგან ორში შესაძლებელია დამზერილ იქნას იისფერი წანაცვლება, მაშინ როდესაც მესამე ექსპერიმენტს შეუძლია გადაჭრას წინააღმდეგობა აინშტაინის მიზიდულობის კანონსა და “დინებადი სივრცის”

პოსტულატს შორის. სამყაროს შორეული ობიექტების წითელი წანაცვლება ახსნილია "გაფართოებადი სამყაროს" შესახებ ჰაბლის ჰიპოთეზაზე დაყრდნობის გარეშე. სტატიის ბოლოს ნაჩვენებია, რომ წარმოდგენილი თეორია შეიძლება დადასტურდეს, თუ განვიხილავთ ზოგიერთი ნაწილაკის ნაკადს (საწინააღმდეგოდ ფარდობითობის ზოგადი თეორიისა, რომელსაც ჰილბერტის გავლენით გააჩნია წმინდა გეომეტრიული, არაფიზიკური აზრი).

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