

They're off!

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Enviado a

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Reconocimiento: la maestra Jovita Lerma Rico condujo parte de las actividades con un grupo de sexto de primaria. Sus observaciones de la reacciones de los alumnos y sus sugerencias contribuyeron a darle forma final de este documento.

Activity 1 Tossing dice: small group activity

Let's simulate a horse race in the following way: 12 horses take part, numbered from 1 to 12. Two dice are tossed and the sum called. The horse with that number advances one place. The first horse to advance 9 places is the winner.

Every student should choose one horse number. Taking turns, toss the dice and call the sum. Fill out the chart showing the progress of each horse.

1	2	3	4	5	6	7	8	9	10	11	12		
													START
													FINISH

Chart 1

Horse 1 is a lousy bet (why?), but any other horse at least has a chance to win.

Activity 2

Compare your results with the results of other groups. Are horses in the middle (6, 7, 8) more likely to win than those on the sides (2, 3, or 11, 12)? Why?

Do you agree in your group that there is a favorite horse, that is, one that has a higher probability to win than the others?

Fill the chart below indicating in how many different ways a sum can be obtained. For example, the sum 4 can be obtained if the dice show 1, 3 or 2, 2 or 3, 1.

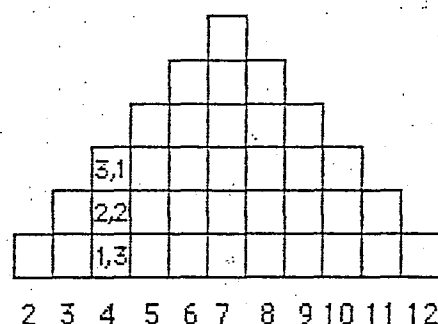


Chart 2

In how many ways can the sum 12 be obtained? _____. In how many ways can the sum 7 be obtained? _____.

According to the chart, what horse would be the favorite?

Does the favorite always win?

Activity 3 Use of random table

Let's simulate the race now using the attached random table.

Notice that in the table appear numbers between 1 and 6, grouped in pairs.

The table simulates the results of tossing many pairs of dice.

To use the table, close your eyes and point with the pencil to some place of the table. Circle the pair of numbers closest to the mark. Afterwards, take the pairs of numbers as they appear on the table.

Sum the two numbers in each pair encountered. Proceede as in the previous

activity, filling out the chart showing the progress of each horse.

1	2	3	4	5	6	7	8	9	10	11	12		
													START
													FINISH

Chart 3

Choose a horse number and run the race. It takes less time using the random table than tossing the dice.

Random tables are used to simulate random processes, without having to actually conduct the random experiment. Random tables can be constructed with the help of computers.

RANDOM TABLE
SIMULATION OF TOSSING 144 PAIRS OF DICE

2 4	3 6	1 4	2 1	2 3	5 2	6 4	1 2
5 2	3 6	1 5	5 4	3 1	5 5	3 4	1 2
1 4	1 1	6 5	2 4	3 1	3 6	4 1	1 5
5 5	2 1	6 6	4 5	4 6	6 3	4 4	3 1
1 2	4 1	4 1	2 6	3 5	5 4	6 5	5 3
5 4	4 3	3 4	3 5	5 6	6 2	6 6	1 4
1 4	1 1	1 2	6 4	2 4	1 6	3 3	4 2
5 6	6 5	6 6	5 3	4 6	3 2	1 1	1 5
3 2	4 1	5 5	6 2	4 2	3 3	3 2	2 6
3 3	6 3	2 2	3 5	5 6	6 2	4 1	4 1
4 2	2 6	3 2	5 2	2 4	1 6	4 2	1 4
1 6	4 1	1 1	1 2	4 2	6 3	5 1	2 2
5 6	5 6	6 1	5 4	4 2	5 1	4 4	4 5
1 2	3 1	4 3	6 1	5 6	3 3	3 3	5 2
2 2	1 1	2 5	3 2	6 6	6 4	4 1	2 2
5 1	3 3	5 1	4 4	5 4	5 6	1 6	3 6
4 4	5 5	5 2	5 3	1 6	4 6	5 2	3 1
5 3	6 5	2 2	2 2	2 4	1 3	4 5	3 4

Activity 4 Computer simulation

Here is a BASIC (Apple II version) program that simulates the race:

```
10 REM THEY'RE OFF!  
20 HOME  
30 DIM V(12)  
40 LET R1 = 1 + INT (6 * RND (1))  
50 LET R2 = 1 + INT (6 * RND (1))  
60 LET S = R1 + R2  
70 LET V(S) = V(S) + 1  
80 VTAB (2*V(S)): HTAB (3*S): PRINT S  
90 IF V(S) = 9 THEN END  
100 GOTO 40
```

Choose a number again. RUN the program. Which horse was the winner?

Run the program several times. Does the favorite horse always win? Is there an unexpected winner (a dark horse) sometimes?

What is the relationship between the shape of Chart 2 and the advancement of different horses?

Keep track of the winners in each race until a horse wins ten races.

2	3	4	5	6	7	8	9	10	11	12

Chart 4 Winners in races of length 9

Write down how many times did each horse win:

Horse: 2 3 4 5 6 7 8 9 10 11 12

Activity 5 Computer simulation

Modify line 90 to make the race shorter.

```
90 IF V(S) = 3 THEN END
```

Would you expect the favorite to win more often, or less often?

Run the program several times. Keep track of winners in each successive race in the following chart until a horse wins ten races.

	2	3	4	5	6	7	8	9	10	11	12

Chart 5 Winners in races of length 3

Write down how many times did each horse win:

Compare the results of this activity with those of activity 4.

Activity 6 Computer simulation

Make the race longer. Change line 90.

```
90 IF V(S) = 20 THEN END
```

What do you expect now? Will the favorite win more often, or less often?

You will have to change line 80 so that you do not run out of space on the screen.

```
80 VTAB (V(S)): HTAB (3*S): PRINT S
```

Run the program several times. Keep track in the following chart of winners in each race until a horse wins ten races.

	2	3	4	5	6	7	8	9	10	11	12

Chart 6 Winners in races of length 20

Write down how many times did each horse win:

Horse: 2 3 4 5 6 7 8 9 10 11 12

Compare the results of this activity with those of the previous activities.

When can you predict with more certainty that a horse in the central lanes (6, 7, 8) will win, when the race is short or longer?

References

Cwirko-Godycki, Jerzy. Mathematical Activities from Poland. Association of Teachers of Mathematics, (no place, no date).

Flores, Alfinio. A microcomputer and the law of small numbers. Arithmetic Teacher, 31(7), March 1984, 60-61.

Flores Peñafiel, A; Lerma Rico, J.; Martínez Cruz, A.; Mirabal G., F. Prácticas de matemáticas para primero de secundaria. Comunicaciones del CIMAT, 1987.

Statistics and information organization. Mathematics Resource Project. Preliminary Edition. Creative Publications, 1978.

Teacher notes

In a random experiment it is impossible to predict the result with absolute certainty. However, if the experiment is conducted many times and the distribution of the results is known, this knowledge can be used to estimate the probability of a desired outcome of the experiment. Since conducting experiments can be time-consuming, a random table or a computer can be used to simulate a great number of experiments.

Activity 1

Although tossing dice is time consuming, it is highly recommended to let children toss the dice at least during one race.

Most children quickly realize that horse 1 will never move, since the smallest possible sum with two dice is $1 + 1 = 2$. However, in the first race usually all other horses are chosen. This activity can also be done with the whole class.

Activity 2

After running several races, students realize that horses in the middle lanes (6, 7, 8) win more often than horses on the sides (2, 3, 11, 12). However it is also important that they realize why.

Although all the sums from 2 to 12 are possible, they are not equally likely. Using two dice, the sum 12 can be obtained in only one way $6 + 6$, but the sum 4 can be obtained in three different ways: $3 + 1$, $2 + 2$, $1 + 3$.

When filling chart 2, children who are not very familiar with dice may have to be remembered that a die has only six faces, numbered 1 through six, so that $1 + 7$ is **not** a possible way to get the sum 8 using two dice.

Since all of the 36 possible combinations of the two dice are equally likely and three of them will give the sum 4 but only one will give the sum 12, in the long run the sum 4 will be called more often than the sum 12.

Chart 2 has precisely 36 cells with all the possible outcomes of two dice.

					1,6									
				1,5	2,5	2,6								
			1,4	2,4	3,4	3,5	3,6							
		1,3	2,3	3,3	4,3	4,4	4,5	4,6						
	1,2	2,2	3,2	4,2	5,2	5,3	5,4	5,5	5,6					
1,1	2,1	3,1	4,1	5,1	6,1	6,2	6,3	6,4	6,5	6,6				
2	3	4	5	6	7	8	9	10	11	12				

Chart 2

Students should realize thus that sum 7 has the greatest probability to be called. However, this fact does not guarantee that in a race the winner will be horse 7.

Another way to list the results is the following table. The outcomes that add to a given sum can be located diagonally, as shown with the sum 6.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Activity 3

The numbers (1 through 6) that appear on the random table were generated by a computer simulating the tossing of a die. Each number between 1 and 6 has the same probability to appear ($1/6$), in every direction of the table, vertically, horizontally etc.

Thus if we want to simulate the tossing of two dice, all we have to do is to read pairs of numbers from the table.

The blind selection of the first pair is recommended to avoid unconscious preferences or avoidances of some numbers. Once the first pair is chosen the next pair should be the next to the left (or to the right, or below or above) to save time. The direction chosen does not matter, but once chosen, students should keep the same procedure.

Random tables are very convenient. A greater number of experiments can be simulated in less time. In some situations random tables are more readily available than computers.

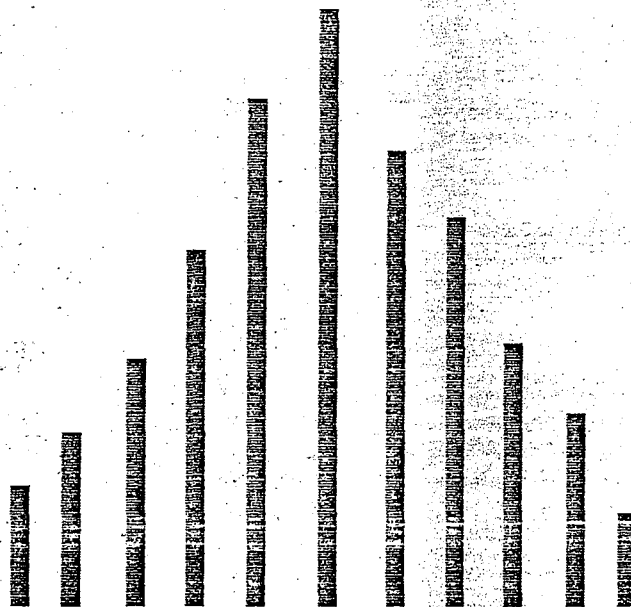
Activity 4

The heart of the computer program used in activity 4 are lines 40-60. Line 40 simulates tossing one die, line 50 tossing another die and line 60 gets the sum.

Notice that the advancement of the horses does resemble chart 2. Horses with a higher probability to move, will usually advance farther. However, the

resemblance is only gross. It can happen that a horse with a smaller probability to move on, does indeed get farther than a horse with a higher probability. Students should not expect regularity in the short run. In order to have an empirical distribution that is more alike to the theoretical distribution, a much-greater number of experiments is usually needed.

The following figure shows the result of a real long race (length 200) drawn with a different scale. Notice that the resemblance to chart 2 is much greater.



When filling chart in this activity, note that winners of races of length 9 are almost-in the five central lanes (5-9).

The program of this activity can be easily modified to automatically simulate race after race with a convenient pause in between for students to note down outcomes. In this way the activity can also be done with the whole class.

Activity 5

Making the race real short emphasizes the fact that with a small number of experiments it is harder to predict the winner. Note that the winners are more spreaded, that is not only horses in the central lanes (5 - 9) win.

Activity 6

Making the race longer will help students to correctly predict the winner. Note that winners are almost exclusively from the three central lanes (6, 7, 8).