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AN OCCUPATION TIME APPROACH FOR CONVERGENCE OF MEASURE-VALUED PROCESSES. APPLICATION TO BRANCHING SYSTEMS

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Abstract

An occupation time approach for weak convergence of measure-valued processes is given, and it is exemplified by showing convergence of branching particle systems to (d, α, β)-superprocesses.

BRANCHING PARTICLE SYSTEM, SUPERPROCESS, OCCUPATION TIME, WEAK CONVERGENCE.

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1 Introduction

The idea of regarding processes as time random fields (which are occupation times) as a way of proving weak convergence was introduced in [3] assuming tightness, and extended in [18] to include tightness. This was done in the context of nuclear space-valued processes. The criteria proved in [3, 18] can be translated directly for measure-valued processes (Theorems 1 and 2). We exemplify the use of Theorem 1 by proving weak convergence of branching particle systems to superprocesses (Theorem 3). For this we use a martingale method inspired in [12, 27, 28].

Existence of (Dawson-Watanabe) superprocesses as weak limits of branching particle systems has been shown by several methods [4, 7, 9, 13, 16, 20, 24, 27, 28, 29]. Other ways of constructing superprocesses are given in [14, 22, 23]. The occupation

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time of a superprocess was first studied in [20] (other relevant papers on the subject are [2, 8, 11, 15, 21]). The occupation time of the particle process is used in [25] in connection with an ergodic result. The recent works of Dawson [5], Dawson and Perkins [7], and Dynkin [10] contain extensive surveys on superprocesses.

Our proof of existence of superprocesses as limits of branching particle systems is intended only to illustrate the occupation time approach by an example, although for this model it is not more efficient than the other known convergence proofs, and the main ingredients are the same. We will explore elsewhere applications of the occupation time approach to derive new information on the branching model.

In the remainder of this section we introduce the notation we need and recall some technical points.

Let $C_c(R^d)_+$ denote the space of non-negative continuous functions on R^d with compact supports, and $K_p(R^d)$ the space of continuous functions φ of the form

$$\varphi = \psi + a\varphi_p, \quad \psi \in C_c(R^d)_+, \quad a \in R_+,$$

where $\varphi_p(x) = (1+ \mid x \mid^2)^{-p}$, $p > 0$, $x \in R^d$. Let $C_p(R^d)$ (resp. $C_p(R^d)_+$) denote the space of continuous (resp. non-negative continuous) functions on R^d such that $\sup_x \mid \varphi(x)/\varphi_p(x) \mid < \infty$. We designate by $\mathcal{M}_p(R^d)$ the space of Radon measures μ on R^d such that $\int \varphi_p \, d\mu < \infty$, equipped with the p-vague topology, i.e. the topology generated by the maps $\mu \mapsto \langle \mu, \varphi \rangle \equiv \int \varphi \, d\mu$ for all $\varphi \in K_p(R^d)$, and by $\mathcal{N}_p(R^d)$ the subspace of $\mathcal{M}_p(R^d)$ of counting measures. If I is a subinterval of R_+, we denote by $C(I)_+$ the space of continuous non-negative functions on I, and by $D(I, \mathcal{M}_p(R^d))$ the space of right-continuous with left limits functions from I into $\mathcal{M}_p(R^d)$, endowed with the Skorokhod topology.

We embed $\mathcal{M}_p(R^d)$ in the locally compact space $\mathcal{M}_p(\dot{R}^d)$, where $\dot{R}^d = R^d \cup \{\tau\}$, τ being an isolated point. $\mathcal{M}_p(\dot{R}^d)$ is the space of non-negative Radon measures μ on $\dot{R}^d$ such that $\int \varphi_p \, d\mu \mid_{R^d} + \mu(\{\tau\}) < \infty$, and the p-vague topology on $\mathcal{M}_p(\dot{R}^d)$ is defined the same way as above taking all φ in $K_p(\dot{R}^d)$, which is defined as $K_p(R^d)$ replacing φ_p by $\dot{\varphi}_p(x) = \varphi_p(x)1_{R^d}(x) + 1_{\{\tau\}}(x)$, $x \in \dot{R}^d$. Let $C_p(\dot{R}^d)$ denote the space of continuous functions on $\dot{R}^d$ such that $\lim_{|x| \to \infty} |\varphi(x)/\varphi_p(x)| = c \in R_+$ and $\varphi(\tau) = c$. The spaces $C_p(\dot{R}^d)_+$ and $D(I, \mathcal{M}_p(\dot{R}^d))$ and their topologies are defined similarly as above. $\langle \mu, \varphi \rangle$ is extended for $\mu \in \mathcal{M}_p(\dot{R}^d)$, $\varphi \in C_p(\dot{R}^d)_+$.

Other notations we will use are the following:

$\Delta_\alpha \equiv -(-\Delta)^{\alpha/2}$: the infinitesimal generator of the spherically symmetric stable process on R^d with exponent $\alpha \in (0,2]$.

$C_c(R_+, E)$: the space of continuous functions $h: R_+ \to E$ with compact support.

$C^1(I, E)$: the space of continuous functions $h: I \to E$ with continuous derivative.

$C_b^n(R_+)$: the space of real-valued continuous functions with bounded continuous derivatives up to order n.

Regarding d, p and α above, we assume $p > d/2$, and in addition $p < (d+\alpha)/2$ if $\alpha < 2$ (see [6, 20] on this condition).

We denote by ϑ_t the translation

$$(\vartheta_t \varphi)(s) = \varphi(s-t), \quad \varphi \in C_c(R_+)_+,$$

and by $\{\varphi_m\}_m \subset C([0,T])_+$ an arbitrary fixed approximation of the Dirac distribution δ_0 such that $\mathrm{supp}\varphi_m \subset [0,T]$ and $\int_0^T \varphi_m(t)\,dt = 1$ for all m.

We assume that all our measure-valued processes are defined on a fixed probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\{\mathcal{F}_t\}_t$ which satisfies the usual conditions. We may take $\Omega = D(I, \mathcal{M}_p(\dot{R}^d))$, ($I = [0,T]$ or R_+), and $\{\mathcal{F}_t\}_t$ the natural right-continuous filtration (P-completed). Then for each $\psi \in C_p(\dot{R}^d)_+$ we denote by $\{\mathcal{F}(\psi)_t\}_t$ the sub-filtration generated by the projection $\Pi_\psi: D(I, \mathcal{M}_p(\dot{R}^d)) \to D(I, R)$ defined by $\Pi_\psi(x) = \langle x, \psi \rangle \equiv \{\langle x(t), \psi \rangle\,,\ t \in I\}$.

$\Rightarrow$ means weak convergence of random elements.

Given $x \in D(R_+, \mathcal{M}_p(\dot{R}^d))$, we define the (continuous linear) functional

$$\langle \mathcal{J}(x), \psi \rangle = \int_0^\infty \langle x(s), \psi(s) \rangle\, ds$$

for any function $\psi \in C_c(R_+, C_p(\dot{R}^d)_+)$. In particular we write $\langle \mathcal{J}(x), \psi \otimes \varphi \rangle$ if $\psi(s)$ is of the form $\psi(s) = \psi \otimes \varphi(s)$, $\psi \in C_p(\dot{R}^d)_+$, $\varphi \in C_c(R_+)_+$. This means that x is regarded as a "time field," and each $\langle \mathcal{J}(x), \psi \rangle$ represents a weighted occupation time of x.

2 Occupation time criteria for weak convergence of measure-valued processes

The following criteria, which we present without proof, are direct translations of the convergence theorems for nuclear space-valued processes proved in [3] and [18]. The only changes are the underlying topological spaces and the use of the tightness theorem of [16], but otherwise the proofs are the same.

Theorem 1. *For each $n \geq 0$, let $X^n \equiv \{X^n(t),\ t \geq 0\}$ be a process with paths in $D(R_+,\ \mathcal{M}_p(\dot{R}^d))$. Assume*

1.1. $\langle \mathcal{J}(X^n), \psi \otimes \varphi \rangle \Rightarrow \left\langle \mathcal{J}\left(X^0\right), \psi \otimes \varphi \right\rangle$ as $n \to \infty$ for all $\psi \in C_p(\dot{R}^d)_+$ and $\varphi \in C_c(R_+)_+$.

1.2. $\{\langle X^n, \psi \rangle\}_{n \geq 1}$ is tight in $D(R_+,\ R)$ for all $\psi \in C_p(\dot{R}^d)_+$.

Then $X^n \Rightarrow X^0$ in $D(R_+,\ \mathcal{M}_p(\dot{R}^d))$ as $n \to \infty$.

Theorem 2. *For each $n \geq 0$, let $X^n \equiv \{X^n(t),\ t \in [0,T]\}$ be a process with paths in $D([0,T],\ \mathcal{M}_p(\dot{R}^d))$. Assume*

2.1. $\langle \mathcal{J}(X^n), \psi \otimes \varphi \rangle \Rightarrow \left\langle \mathcal{J}\left(X^0\right), \psi \otimes \varphi \right\rangle$ as $n \to \infty$ for all $\psi \in C_p(\dot{R}^d)_+$ and $\psi \in C([0,T])_+$.

2.2. $\lim_{m \to \infty} \limsup_{n \to \infty} \mathrm{P}\left[|\langle \mathcal{J}(X^n), \psi \otimes \vartheta_t \varphi_m \rangle - \langle X^n(t), \psi \rangle| \geq \epsilon\right] = 0$ for all $\psi \in C_p(\dot{R}^d)_+$, $t \in [0,T)$ and $\epsilon > 0$.

2.3. $\lim_{n \to \infty} \limsup_{m \to \infty} \mathrm{P}\left[|\langle \mathcal{J}(X^n), \psi \otimes (\vartheta_{T_n+\gamma_n} - \vartheta_{T_n}) \varphi_m \rangle| \geq \epsilon\right] = 0$ for all $\psi \in C_p(\dot{R}^d)_+$, $(T_n)_n$ sequence of $\mathcal{F}(\psi)_t$-stopping times in $[0,T]$, $(\gamma_n)_n \subset (0,T]$ such that $\gamma_n \to 0$, and $\epsilon > 0$.

Then $X^n \Rightarrow X^0$ in $D([0,T],\ \mathcal{M}_p(\dot{R}^d))$ as $n \to \infty$.

Remarks

(a). Hypotheses 2.1 and 2.2 together imply weak convergence of finite-dimensional distributions. Hypothesis 2.3 is equivalent to the stopping time condition of Aldous

[1]. When the limit process is continuous, the hypotheses 2.1, 2.2 and 2.3 are necessary (see [18]).

(b). In both theorems it suffices to take test functions ψ and φ in dense subsets.

3 Convergence of branching particle systems to superprocesses via the occupation time

We recall first the minimum necessary background on superprocesses, their occupation times and the approximating particle systems. We restrict ourselves to the class of (d,α,β)-superprocesses [5], but the method can be used more generally. (Recall that we assume $p > d/2$, and in addition $p < (d+\alpha)/2$ in case $\alpha < 2$).

The (d,α,β)-*superprocess* $X \equiv \{X(t),\ t \geq 0\}$ is a homogeneous Markov process with paths in $D(R_+, \mathcal{M}_p(R^d))$, whose transition Laplace functional is given by

$$E\left[e^{-\langle X(t),\psi\rangle}\Big| X(0) = \mu\right] = e^{-\langle \mu, u_\psi(t)\rangle}, \quad \psi \in C_p(R^d)_+, \quad \mu \in \mathcal{M}_p(R^d),$$

where u_ψ is the unique, global, non-negative (mild) solution of the non-linear equation

$$\begin{aligned} \frac{\partial u(t)}{\partial t} &= (\Delta_\alpha + Vb)\,u(t) - Vc\,(u(t))^{1+\beta}, \\ u(x,0) &= \psi(x), \quad \psi \in C_p(R^d)_+, \end{aligned}$$

with constants $\alpha \in (0,2]$, $V > 0$, $b \in R$, $c \in (0, 1/(1+\beta)]$ and $\beta \in (0,1]$.

The *occupation time process* of X is the $\mathcal{M}_p(R^d)$-valued process $Y \equiv \{Y(t),\ t \geq 0\}$ defined by

$$\langle Y(t),\psi\rangle = \int_0^t \langle X(s),\psi\rangle\, ds, \quad \psi \in C_p(R^d)_+, \quad t \geq 0.$$

More generally, looking at X as a "time random field" we define the space-time random field $\mathcal{J}(X)$ by

$$\langle \mathcal{J}(X), \psi\rangle = \int_0^\infty \langle X(s), \psi(s)\rangle\, ds,$$

for any $\psi \in C_c\left(R_+, C_p(R^d)_+\right)$.

For fixed $T > 0$, $\psi \in C_p(R^d)_+$ and $\varphi \in C([0,T])_+$, $\mathcal{J}(X)$ is characterized by its conditional Laplace functional, given by

$$E\left[e^{-\langle \mathcal{J}(X), \psi\otimes\varphi\rangle}\Big| X(0) = \mu\right] = e^{-\langle \mu, u_0(T)\rangle},$$

where $u_\xi \equiv u_{\xi,\psi,\varphi}$ is the unique (mild) solution of the non-linear equation

$$\begin{aligned} \frac{\partial u_\xi(t)}{\partial t} &= (\Delta_\alpha + Vb)\, u_\xi(t) - Vc\,(u_\xi(t))^{1+\beta} + \psi\varphi(T-t), \quad 0 \le t \le T, \qquad (3.1) \\ u_\xi(x,0) &= \xi(x), \quad \xi \in C_p(R^d)_+. \end{aligned}$$

[8, 11, 12, 20, 28]. The solution is classical if $\xi \in \mathrm{Dom}\,(\Delta_\alpha)$ and $\varphi \in C([0,T])_+ \cap C^1([0,T])$.

The approximating particle system is constructed as follows. At time 0 the particles are distributed by a random point measure on R^d, and then they independently migrate according to Δ_α during V-exponentially distributed lifetimes, at the end of which they produce particles according to the generating function

$$F(s) = s + b(s-1) + c(1-s)^{1+\beta}, \quad s \in [0,1],$$

and the new particles evolve in the same manner, starting from the death site of their parent. (The fact that F is the generating function of a branching law requires $b \in (-1, c]$, $c \in (0, (1+b)/(1+\beta)]$).

The *particle process* $N \equiv \{N(t),\ t \ge 0\}$ is defined by $N(t) = \sum_i \delta_{x_i(t)}$, where $\{x_i(t)\}_i$ are the locations of the particles present at time t. The process N is homogeneous Markov with paths in $D(R_+, \mathcal{M}_p(R^d))$ and transition Laplace functional given by

$$E\left[e^{-\langle N(t),\psi\rangle}\middle| N(0)=\mu\right] = e^{\langle \mu, \log(1-v_\psi(t))\rangle}, \quad \psi \in C_p(R^d)_+, \quad \mu \in \mathcal{N}_p(R^d),$$

where v_ψ is the unique, global, non-negative (mild) solution of the non-linear equation

$$\begin{aligned} \frac{\partial v(t)}{\partial t} &= (\Delta_\alpha + Vb)\, v(t) - Vc\,(v(t))^{1+\beta}, \\ v(x,0) &= 1 - e^{-\psi(x)}, \quad \psi \in C_p(R^d)_+. \end{aligned}$$

Note that this is the same non-linear equation as for the superprocess, but the initial condition is different.

The rescaling of the particle process which yields the superprocess is as follows. In the n-th rescaling the parameters V and b are $V_n = Vn^\beta$ and $b_n = bn^{-\beta}$, and if N^n designates the corresponding particle process, then $X^n = n^{-1}N^n$ denotes the mass process obtained by giving each particle a mass n^{-1}. We then have the following well-known basic result (see e.g. [5]).

Theorem 3. *If $X^n(0) \Rightarrow X(0)$ as $n \to \infty$, then $X^n \Rightarrow X$ in $D(R_+, \mathcal{M}_p(\dot{R}^d))$ as $n \to \infty$.*

We will prove Theorem 3 by means of Theorem 1. We may take a fixed time interval $[0,T]$. Hence we must work with $\mathcal{J}(X^n)$ defined by

$$\langle \mathcal{J}(X^n), \psi \rangle = \int_0^T \langle X^n(s), \psi(s) \rangle \, ds, \quad \psi \in C([0,T], C_p(\dot{R}^d)_+).$$

Proof. Since we already know tightness [5], it suffices to verify hypothesis 1.1. (Tightness is not the main point here. It could be proved using Theorem 2, but the calculations would be basically the same as in [5]). From the Markov property of X^n we know that for any $f \in C_b^3(R_+)$, $\psi \in C_c(R_+, \mathrm{Dom}(\Delta_\alpha)) \cap C^1(R_+, C_p(R^d)_+)$, the process

$$\begin{aligned} M^n(t) &= f(\langle X^n(t), \psi(t) \rangle) \\ &\quad - \int_0^t \left[f'(\langle X^n(s), \psi(s) \rangle) \left\langle X^n(s), \tfrac{\partial}{\partial s}\psi(s) \right\rangle + A^n f(\langle X^n(s), \psi(s) \rangle) \right] ds, \\ &\quad t \geq 0, \end{aligned}$$

is a martingale, where

$$\begin{aligned} A^n f(\langle \mu, \psi \rangle) &= \Delta_\alpha f(\langle \mu, \psi \rangle) \\ &\quad + n^{1+\beta} V \left\langle \mu, \sum_{k=0}^{\infty} p_k^n \left[f\left(\langle \mu, \psi \rangle + n^{-1}(k-1)\psi \right) - f(\langle \mu, \psi \rangle) \right] \right\rangle, \\ &\quad \psi \in \mathrm{Dom}(\Delta_\alpha), \quad n\mu \in \mathcal{N}_p(R^d), \end{aligned}$$

and $(p_k^n)_k$ denotes the branching law with generating function

$$F^n(s) = s + bn^{-\beta}(s-1) + c(1-s)^{1+\beta}.$$

Taking $f(x) = e^{-x}$ we have

$$A^n e^{-\langle \mu, \psi \rangle} = \Delta_\alpha e^{-\langle \mu, \psi \rangle} + n^{1+\beta} V \left\langle \mu, e^{\psi/n} \left(F^n\left(e^{-\psi/n}\right) - e^{-\psi/n} \right) \right\rangle e^{-\langle \mu, \psi \rangle},$$

and

$$\Delta_\alpha e^{-\langle X^n(t), \psi \rangle} = e^{-\langle X^n(t), \psi \rangle} \left\langle X^n(t), e^{\psi/n} \Delta_\alpha \left(n e^{-\psi/n} \right) \right\rangle.$$

Hence, denoting $h(T-t) = \psi(t)$, it follows that

$$M^n(t) = e^{-\langle X^n(t), h(T-t) \rangle}$$

$$
\begin{aligned}
&-\int_0^t \Big\{ -e^{-\langle X^n(s), h(T-s)\rangle} \Big\langle X^n(s), -\tfrac{\partial}{\partial (T-s)} h(T-s) \\
&-e^{h(T-s)/n} \Delta_\alpha \left(n e^{-h(T-s)/n} \right) \\
&-n^{1+\beta} V e^{h(T-s)/n} \left[F^n \left(e^{-h(T-s)/n} \right) - e^{-h(T-s)/n} \right] \Big\rangle \Big\} \, ds \\
&0 \le t \le T,
\end{aligned}
$$

is a martingale.

We now fix $\Psi \in C_p(R^d)_+$ and $\Phi \in C([0,T])_+ \cap C^1([0,T])$, and assume that $h^n(t) \equiv h^n_{\Psi,\Phi}(t)$ satisfies the equation

$$
\begin{aligned}
\frac{\partial h^n(t)}{\partial t} &= -e^{h^n(t)/n} \Delta_\alpha \left(n e^{-h^n(t)/n} \right) \\
&\quad -n^{1+\beta} V e^{h^n(t)/n} \left[F^n \left(e^{-h^n(t)/n} \right) - e^{-h^n(t)/n} \right] \\
&\quad +\Psi \Phi (T-t), \quad 0 \le t \le T, \\
h^n(0) &= 0,
\end{aligned}
$$

in the classical sense, or equivalently, $u^n(t) \equiv u^n_{\Psi,\Phi}(t) = n\left(1 - e^{-h^n(t)/n}\right)$ solves the equation

$$
\begin{aligned}
\frac{\partial u^n(t)}{\partial t} &= (\Delta_\alpha + Vb)\, u^n(t) - Vc(u^n(t))^{1+\beta} \\
&\quad + (1 - u^n(t)/n)\, \Psi \Phi(T-t), \quad 0 \le t \le T, \qquad (3.2) \\
u^n(0) &= 0.
\end{aligned}
$$

The existence of a unique classical solution of (3.2) follows from Theorem 1.5, Chapter 6, of [26]. Then

$$
M^n(t) = e^{-\langle X^n(t), h^n(T-t)\rangle} - \int_0^t e^{-\langle X^n(s), h^n(T-s)\rangle} \langle X^n(s), \Psi \rangle \, \Phi(s)\, ds, \quad 0 \le t \le T,
$$

is a martingale, and consequently, by Corollary 3.3, Chapter 2, of [13],

$$
Z^n(t) = \exp\left\{ -\langle X^n(t), h^n(T-t)\rangle - \int_0^t \langle X^n(s), \Psi\rangle \, \Phi(s)\, ds \right\}, \quad 0 \le t \le T,
$$

is a (local) martingale. Hence, since $EZ^n(T) = EZ^n(0)$, we obtain

$$
E \exp\left\{ -\int_0^T \langle X^n(s), \Psi \rangle \, \Phi(s)\, ds \right\} = E \exp\left\{ \langle X^n(0), n \log(1 - u^n(T)/n) \rangle \right\}. \qquad (3.3)
$$

Now, $u^n(t) \to u(t)$ as $n \to \infty$, uniformly in t, where $u(t) \equiv u_{\Psi,\Phi}(t)$ is the solution of

$$\begin{aligned} \frac{\partial u(t)}{\partial t} &= (\Delta_\alpha + Vb)\, u(t) - Vc(u(t))^{1+\beta} + \Psi\Phi(T-t), \quad 0 \le t \le T, \\ u(0) &= 0. \end{aligned}$$

This can be proven by a continuous dependence theorem (e.g., Theorem 3.4.1 of [19]), or directly, by showing that $\{u_n\}_n$ is relatively compact in $C([0,T], C_p(R^d)_+)$, because then every limit point is the unique solution of (3.1) with $\xi = 0$. Therefore, since $X^n(0) \Rightarrow X(0)$ as $n \to \infty$ by assumption, from (3.3) we have

$$E \exp\left\{ - \int_0^T \langle X^n(s), \Psi\rangle\, \Phi(s)\, ds \right\} \to E\exp\{-\langle X(0), u(T)\rangle\} \text{ as } n \to \infty.$$

But we know that for the superprocess X,

$$E \exp\left\{ - \int_0^T \langle X(s), \Psi\rangle\, \Phi(s)\, ds \right\} = E\exp\{-\langle X(0), u(T)\rangle\}.$$

Hence we conclude that

$$\langle \mathcal{J}(X^n), \Psi \otimes \Phi \rangle \Rightarrow \langle \mathcal{J}(X), \Psi \otimes \Phi \rangle \text{ as } n \to \infty$$

for all $\Psi \in C_p(R^d)_+$, $\Phi \in C([0,T])_+ \cap C^1([0,T])$.

Remark. From (3.2) and (3.3) for $n = 1$ we obtain a characterization of the occupation time of the particle process N by its Laplace functional:

$$E\left[\exp\left\{ - \int_0^T \langle N(s), \psi\rangle\, \varphi(s)\, ds \right\} \middle| N(0) = \mu \right] = \exp\left\{ \left\langle \mu, \log\left(1 - u(T)\right)\right\rangle \right\},$$

$$\psi \in C_p(R^d)_+,\ \varphi \in C([0,T])_+,\ \mu \in \mathcal{N}_p(R^d),$$

where $u(t) \equiv u_{\psi,\varphi}(t)$ is the unique solution of the non-linear equation

$$\begin{aligned} \frac{\partial u(t)}{\partial t} &= (\Delta_\alpha + Vb)\, u(t) - Vc(u(t))^{1+\beta} + (1-u(t))\psi\varphi(T-t), \ 0 \le t \le T, \\ u(0) &= 0. \end{aligned}$$

Putting $\varphi \equiv 1$, this coincides with Proposition 1 of [25].

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