MINIMIZING THE RUIN PROBABILITY OF RISK PROCESSES WITH REINSURANCE

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Abstract

For two combinations of proportional and excess of loss reinsurance in a renewal risk process, we investigate existence of the insurer's adjustment coefficient as a function of retention levels, assuming that the premiums are calculated according to the expected value principle. In the classical Poisson compound case with exponentially distributed claims we prove, under some additional assumptions, unimodality of the adjustment coefficient as a function of the retention levels. For the maximal adjustment coefficient the ruin probability is minimal. Our results complement previous work of Waters [8], Centeno [3] and Hesselager [4].

AMS Subject Classification: 91B30, 91B70

Key Words and Phrases: Classical Risk Processes; Adjustment coefficient, Reinsurance; Combinations of excess of loss and proportional reinsurance

1 Introduction and Background Results

In several papers and textbooks on optimal reinsurance, it is assumed that the insurer estimates the probability of ruin using the Cramer-Lundberg approximation (if the adjustment coefficient exists). Thus, as the adjustment exponent increases, the ruin probability decreases exponentially fast. The effects of reinsurance treaties on the Cramer-Lundberg coefficient have been investigated in various papers (see e.g. Waters (1979,1983), Hesselager (1990), Kaluszka (2001), Centeno (1985,1986,2002), and the references therein). Assuming that the reinsurer premiums are calculated according to the standard deviation and variance principles, Hesselager (1990) studied optimal reinsurance treaties when both, the insurer and the reinsurer, aim to minimize their ruin probabilities. He considered three different types f^A, f^B and f^C, of reinsurance compensation functions, namely

$$\begin{aligned} f^A(x) &= ax + (1-a)\max\{0, x-M\} \\ f^B(x) &= \min\{ax, \max\{0, x-M\}\} \\ f^C(x) &= a\max\{0, x-M\}, \end{aligned}$$

where $a \in [0,1]$ and $M \geq 0$ are fixed parameters of the model, termed retention levels. Hesselager proved that for all global and individual reinsurance treaties with general Vajda compensation function f, the corresponding reinsurer's adjustment coefficient R_f fulfills

$$R_{f^A} \leq R_f \leq R_{f^B},$$

and if f is convex, then

$$R_{f^A} \leq R_f \leq R_{f^C}.$$

As a consequence, the ruin probability for the reinsurer is minimal for reinsurance treaties B and C, respectively.

Centeno (2002) investigated reinsurance treaties of type A for renewal risk models. In the case when the proportional reinsurance premium is calculated on original terms, and the excess of loss premium is calculated according to the expected value principle, she proved that the insurer's adjustment coefficient has a unique maximum with respect to the retention levels a and M (this property of the coefficient is called unimodality).

Following the approach of Centeno (2002), in this paper we investigate reinsurance treaties of types B and C. Assuming that the premiums are calculated according to the expected value principle, we obtain conditions for existence of the insurer's adjustment coefficient as a function of levels M and a. In the case of classical risk processes with exponentially distributed claims, under some restrictions on the reinsurance levels a, M and the premium c, we also prove unimodality of the adjustment coefficient.

The general renewal risk model is described as follows (for more details see e.g. Rolski et all. [6]). The number of claims $N(t)$ arriving at a insurance company in the time interval $[0,t]$ is given by $N(t) = \sup\{n : S_n \leq t\}$, where $S_0 = 0$, $S_n = T_1 + T_2 + \cdots + T_n$, and T_n is the interarrival time between the $(n-1)$th and the nth claims. We suppose that $\{T_n\}_{n=1}^{\infty}$ is a sequence of independent and identically distributed random variables with mean value $1/\gamma$.

Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of non-negative, independent and identically distributed random variables, having common distribution function F with mean μ and density function f. Here X_n corresponds to the amount of nth claim. We assume that the moment generating function $M_X(r) = E[e^{rX_i}]$ exists on $(-\infty, \tau)$ for some $0 < \tau \leq \infty$. We also assume that $\{X_n\}_{n=1}^{\infty}$ is independent of $\{T_n\}_{n=1}^{\infty}$, and that F is strictly increasing in $[0,\infty)$ with $F(0) = 0$. This implies $0 < F(x) < 1$ for all $x > 0$. The risk process $\{Y_t,\ t \geq 0\}$ is defined by

$$Y(t) = ct - \sum_{n=1}^{N(t)} X_n,$$

where $c > 0$ is the insurer's premium income per unit time. The classical risk process corresponds to exponentially distributed interarrival times. The ruin probability $\psi(u)$ associated to an initial capital $u \geq 0$ is defined by

$$\psi(u) = \inf\{t > 0 : Y(t) < 0\}.$$

We suppose that our model satisfies the positive safety loading condition

$$c > \lambda\mu, \tag{1}$$

which yields $\psi(u) < 1$ for each $u \geq 0$.

Let $Y_n = X_n - cT_n$, $n = 1, 2, \ldots$, and let $g(r) = Ee^{rY_n}$ be the moment generating function of Y_n. The adjustment coefficient R of the risk process $\{Y(t),\ t \geq 0\}$ is defined as the unique positive solution to the equation $g(r) = 1$, when such solution exists, and 0 otherwise, and satisfies the Cramer-Lundberg inequality

$$\psi(u) \leq e^{-Ru}, \quad u \geq 0, \tag{2}$$

and the Cramer-Lundberg approximation

$$\psi(u) \approx Ce^{-Ru}, u \to \infty, \tag{3}$$

for some constant C.

Because of the last inequality, the adjustment coefficient is considered a measure of riskiness of $\{Y(t),\ t \geq 0\}$. Hence, maximization of R as a function of the parameters of the process is relevant; we will consider the important case in which the insurance company takes a reinsurance contract.

2 Main Results and Proofs

We suppose that the insurer has the choice of reinsuring risk by reinsurance treaties of type B and C. This means that the reinsurer retains in the n^{th} claim the amounts

$$Z_n^B = \min\{aX_n, \max(0, X_n - M)\},$$

and

$$Z_n^C = a\max\{(0, X_n - M)\},$$

respectively, and the reinsurer covers the difference $X_n - Z_n$. We will suppose that the retention limits a and M are real numbers satisfying $M \geq 0$ and

$0 \leq a < 1$. (The case $a = 1$ is a particular instance of the reinsurance model studied by Centeno (2002)).

We also assume that the corresponding reinsurance premiums $P^B_{a,M}$ and $P^C_{a,M}$ are calculated according to the expected value principle with loading coefficient α, i.e.,

$$\begin{aligned}
&P^B_{a,M} \\
&\quad = (1+\alpha)\gamma E[Z^B_n] = (1+\alpha)\gamma E[\min\{aX_n, \max(0, X_n - M)\}] \\
&\quad = (1+\alpha)\gamma \left[\int_M^{\frac{M}{1-a}} x f(x)\, dx - M \left[F(\frac{M}{1-a}) - F(M) \right] + \int_{\frac{M}{1-a}}^{\infty} a x f(x) dx \right],
\end{aligned}$$

and

$$P^B_{a,M} = (1+\alpha)\gamma E[Z^B_n] = (1+\alpha)\gamma \left[\int_M^{\infty} (x - M) f(x)\, dx \right].$$

Thus, for treaties of type B, the insurer's adjustment coefficient $R_{a,M}$ is the unique positive root of

$$g_{a,M}(r) := E[e^{r(X_n - Z^B_n) - crT + rTP^B_{a,M}}] = 1 \tag{4}$$

if such solution exists, and 0 otherwise. The adjustment coefficient for treaties of type C are defined similarly. Since

$$\psi(u) \approx Ce^{-R_{a,M}u}$$

for big u by (3), a natural question is to determine whether the adjustment coefficient is a unimodal function of the parameters a and M and what is its maximum.

In addition to (1), let us assume that

$$(1+\alpha)\gamma\mu > c. \tag{5}$$

This condition reflects the fact that the insurer cannot insure the whole risk.

Let us denote by $W(a, M)$ the insurer's net profit per period of time after reinsurance. We write

$$a_0 = \frac{c - \gamma\mu}{\alpha\gamma\mu}$$

and put

$$A = \{a : 0 \leq a < 1, \text{ and there exists } M \geq 0 \,\text{with}\, EW(a, M) = 0.\}$$

From (1) and (5) it follows that $0 < a_0 < 1$. Notice that

$$\begin{aligned}
EW(a, M) &= c - (1+\alpha)\gamma E Z_n - \gamma E(X_n - Z_n) \\
&= c - \alpha\gamma E Z_n - \gamma\mu.
\end{aligned}$$

Let L be the set of points (a, M) for which the expected profit is strictly positive, namely

$$L = \{(a, M) : 0 \leq a < 1,\ M \geq 0,\ E\left[W\left(a, M\right)\right] > 0\}.$$

The following result can be proved similarly as in Centeno (2002).

Lemma 1. *For both types of reinsurance treaties B and C, the adjustment coefficient $R_{a,M}$ is positive if and only if $(a, M) \in L$.*

Our first result is the following theorem.

Theorem 1. *Under conditions (1) and (5), for both types of reinsurance treaties B and C,*

a) $A = [a_0, 1)$.

b) For all $a \in A$ there exists a unique $M \geq 0$ such that $E\left[W\left(a, M\right)\right] = 0$. Let us denote this dependence function of M on a by $\Phi(a)$, where $\Phi : A \to [0, \infty)$. Then for $M > \Phi(a)$ we have that $R_{a,M} > 0$.

c) Φ has continuous first derivative.

d) $\Phi(a_0) = 0$.

Proof. Let $1 \geq a > a_0$. Due to condition (5) we have

$$E\left[W\left(a, 0\right)\right] = c - \alpha\gamma a\mu - \gamma\mu < 0$$

for both types of reinsurance treaties B and C. The safety loading condition gives

$$\lim_{M\to\infty} E\left[W\left(a, M\right)\right] = c - \gamma\mu > 0.$$

Since $EW\left(a, \cdot\right)$ is continuous, the equation

$$E\left[W\left(a, \cdot\right)\right] = 0 \tag{6}$$

has at least one positive solution. In case of reinsurance treaty B

$$\frac{\partial E\left[W\left(a, M\right)\right]}{\partial M} = \alpha\gamma\left[F\left(\frac{M}{1-a}\right) - F\left(M\right)\right] > 0, \tag{7}$$

whereas in case of reinsurance treaty C,

$$\frac{\partial E\left[W\left(a, M\right)\right]}{\partial M} = a\alpha\gamma(1 - F(M)) > 0, \tag{8}$$

which shows that the solution to (6) is unique. Moreover, there is no $M \geq 0$ satisfying $E\left[W\left(a, M\right)\right] = 0$ for $0 \leq a < a_0$. This follows from (7), (8), and the inequalities $EW(a, 0) > 0$, $0 \leq a < a_0$. Using again (7), (8) and that $EW\left(a_0, 0\right) = 0$, we obtain a) and b). Part c) follows easily from the explicit function theorem, and d) is a consequence of c) and the fact that $EW(a_0, 0) = 0$. $\square$

The study of unimodality of $R_{a,M}$ as an implicit solution of (4) can be complicated for general distributions of T and X_n. Here we consider the particular case of a classical risk process with exponentially distributed claims. Without loss of generality, we can assume that $\mu = 1$.

We have the following results on the unimodality of the adjustment coefficient.

Theorem 2. *Consider a classical risk process with exponentially distributed claims and reinsurance treaty of type B. Let $a \geq a_0$, and we assume*

$$\begin{aligned} &(1+\alpha)\gamma > c > \gamma > 1, \\ &M > \max\left\{\alpha, \frac{\gamma \ln[(1+\alpha)\gamma]}{\gamma - 1}\right\}. \end{aligned} \tag{9}$$

Then, the insurer's adjustment coefficient $R_{a,M}$ is a unimodal function of M, and attains its maximum at the point $R' = M^{-1}\ln[(1+\alpha)\gamma]$. In particular, the maximal adjustment coefficiet coincides with the maximal adjustment coefficient obtained by Centeno [3] in the case of the classical risk model with exponentially distributed claims and reinsurance treaty of type A.

Proof. We have

$$\begin{aligned} P_{a,M} &= (1+\alpha)\gamma E[\min\{aX_n, \max(0, X_n - M)\}] \\ &= (1+\alpha)\gamma\left[e^{-M} - (1-a)e^{-\frac{M}{1-a}}\right] \end{aligned}$$

and

$$E[e^{rTP_{a,M} - crT}] = \frac{1}{1 + cr - r(1+\alpha)\gamma\left[e^{-M} - (1-a)e^{-\frac{M}{1-a}}\right]}. \tag{10}$$

Calculating the moment generating function of $X_n - Z_n$ we obtain that it exists for $r < \frac{1}{1-a}$, and for $r \neq 1$,

$$E[e^{r(X_n - Z_n)}] = \frac{e^{(r-1)M}}{r-1} - \frac{1}{r-1} - e^{\frac{M(r-ra-1)}{1-a}} + e^{M(r-1)} - \frac{1}{r-ra-1}e^{\frac{M(r-ra-1)}{1-a}}. \tag{11}$$

Let us note that for $a \geq a_0$, and M satisfying (9), we have $E(W(a,M) > 0$, that is

$$e^{-M} - (1-a)e^{-\frac{M}{1-a}} < \frac{c-\gamma}{\alpha\gamma}, \tag{12}$$

hence the adjustment coefficient $R_{a,M}$ exists and is positive, from Theorem 1. Substituting (10) and (11) into (4), we obtain that $R_{a,M}$ satisfies the equation

$$A(a, R_{a,M}, M) = 0, \tag{13}$$

where

$$\begin{aligned} A(a,r,M) \quad := \quad & (r-ra-1)e^{(r-1)M} \\ & -(1+\alpha)\gamma(r-1)(r-ra-1)[e^{-M}-(1-a)e^{-\frac{M}{1-a}}] \\ & -(r-1)(1-a)e^{\frac{M(r-ra-1)}{1-a}}-(r-ra-1)[1+(r-1)c]. \end{aligned}$$

We will show that the solution $R_{a,M}$ is a unimodal function of M.

Indeed, the implicit function theorem gives

$$\frac{d}{dM}R_{a,M} = -\frac{(d/dM)A(a,r,M)}{(d/dr)A(a,r,M)}|_{r=R_{a,M}}.$$

Since

$$\begin{aligned} \frac{d}{dM}A(a,r,M) \quad = \quad & (r-1)(r-ra-1)[e^{(r-1)M} \\ & +(1+\alpha)\gamma(-e^{-M}+e^{-\frac{M}{1-a}})-e^{M(r-\frac{1}{1-a})}] \\ = \quad & (r-1)(r-ra-1)[e^{rM}-(1+\alpha)\gamma][e^{-M}-e^{-\frac{M}{1-a}}], \end{aligned}$$

the function $R_(a,M)$ has a unique possible inflection point, given by

$$R' = \frac{\ln[(1+\alpha)\gamma]}{M}. \tag{14}$$

We will prove that $\frac{d^2A(a,r,M)}{dM^2}|_{r=R'} < 0$, thus showing that $R_{a,M}$ is a unimodal function of M, attaining its maximum at R'.

Differentiating $R_{a,M}$ twice with respect to M at the point R', we obtain from the implicit function theorem that

$$\frac{d^2R_{a,M}}{dM^2}|_{r=R'} = -\frac{(d^2/dM^2)A(a,r,M)}{(d/dr)A(a,r,M)}|_{r=R'}.$$

Calculating the derivatives in this expression, we get

$$\frac{d^2A(a,r,M)}{dM^2}|_{r=R'} = (1+\alpha)\gamma r(e^{-M}-e^{-\frac{M}{1-a}}) > 0.$$

Calculating the derivative $(d/dr)A(a,r,M)$, we have

$$\begin{aligned} \frac{dA(a,r,M)}{dr} \quad = \quad & (1-a)e^{(r-1)M}+M(r-ra-1)e^{(r-1)M} \\ & -(1+\alpha)\gamma(r-ra-1)[e^{-M}-(1-a)e^{-\frac{M}{1-a}}] \\ & -(1+\alpha)\gamma(r-1)(1-a)[e^{-M}-(1-a)e^{-\frac{M}{1-a}}] \\ & -(1-a)e^{M(r-\frac{1}{1-a})}-(r-1)Me^{M(r-\frac{1}{1-a})} \\ & -(1-a)(1+rc-c)-c(r-ra-1). \end{aligned}$$

Substituting (14) in the previous equation and using (12), we obtain

$$\begin{aligned}
&(d/dr)A(a,r,M)|_{r=R'} \\
&= (1+\alpha)\gamma(1-a)e^{-M} + (1+\alpha)\gamma(R'-R'a-1)Me^{-M} \qquad (15)\\
&\quad -(1+\alpha)\gamma(R'-R'a-1)[e^{-M}-(1-a)e^{-\frac{M}{1-a}}] \\
&\quad -(1+\alpha)\gamma(R'-1)(1-a)[e^{-M}-(1-a)e^{-\frac{M}{1-a}}] \\
&\quad -(1+\alpha)\gamma(1-a)e^{-\frac{M}{1-a}} - (1+\alpha)M\gamma(R'-1)e^{-\frac{M}{1-a}} \\
&\quad -(1-a)(1+R'c-c) - c(R'-R'a-1) \\
&> (1+\alpha)\gamma(1-a)e^{-M} + (1+\alpha)\gamma(R'-R'a-1)Me^{-M} \\
&\quad -(1+\alpha)\gamma(R'-R'a-1)\frac{c-\gamma}{\alpha\gamma} \\
&\quad -(1+\alpha)\gamma(R'-1)(1-a)\frac{c-\gamma}{\alpha\gamma} \\
&\quad -(1+\alpha)\gamma(1-a)e^{-\frac{M}{1-a}} - (1+\alpha)\gamma(R'-1)Me^{-\frac{M}{1-a}} \\
&\quad -(1-a)(1+R'c-c) - c(R'-R'a-1).
\end{aligned}$$

Further, using $e^{-M} > e^{-\frac{M}{1-a}}$, and $R' < 1$, it holds

$$\begin{aligned}
&(d/dr)A(a,r,M)|_{r=R'} \\
&> (1+\alpha)\gamma(-R'+R'a+1)[-Me^{-M} + \frac{c}{(1+\alpha)\gamma)}] - (1-a)(1+R'c-c) \\
&> 0,
\end{aligned}$$

where in the last inequality we used our assumptions on c, M and γ, to obtain that both addends are positive. □

Theorem 3. *Consider a classical risk process with exponentially distributed claims and reinsurance treaty of type C. Let $a \geq a_0$. Under assumptions (1) and (5), there exists some positive constant M_0, such that for all $M > M_0$, the insurer's adjustment coefficient $R_{a,M}$ is a unimodal function of M, and attains its maximum at the unique point satisfying*

$$M = \frac{1}{R_{a,M}} \ln[\alpha(1 + R_{a,M}a - R_{a,M})].$$

Proof. We proceed similarly as in the proof of Theorem 2. We have

$$P_{a,M} = (1+\alpha)\gamma a E[\max(0, X_n - M)] = (1+\alpha)\gamma a e^{-M},$$

and

$$E[e^{rTP_{(a,M)}-crT}] = \frac{1}{1+cr-r(1+\alpha)\gamma a e^{-M}} \qquad (16)$$

for all r satisfying

$$1 + cr - r(1+\alpha)\gamma a e^{-M} > 0.$$

The moment generating function of $X_n - Z_n$ exists for $r < \frac{1}{1-a}$, and is given for $r \neq 1$ by

$$E[e^{r(X_n-Z_n)}] = -\frac{1}{r-1} - \frac{rae^{M(r-1)}}{(r-1)(r-ra-1)}. \tag{17}$$

Substituting (16) and (17) into (4) yields that for fixed $a \geq a_0$, $M \geq \Phi(a)$, the adjustment coefficient $R_{a,M}$ exists and solves the equation

$$-\frac{1}{r-1} - \frac{rae^{M(r-1)}}{(r-1)(r-ra-1)} = 1 + cr - r(1+\alpha)\gamma ae^{-M},$$

which is equivalent to

$$A(a,M,r) := (r-1)(r-ra-1)[1+cr-(1+\alpha)are^{-M}]+rae^{M(r-1)}+r-ra-1 = 0. \tag{18}$$

From the implicit function theorem we obtain

$$\frac{d}{dM}R_{a,M} = -\frac{(d/dM)A(a,M,r)}{(d/dr)A(a,M,r)}|_{r=R_{a,M}}.$$

The possible inflection points R'' of $R_{a,M}$ satisfy $(d/dM)A(a,M,r)|_{r=R''} = 0$. Since

$$(d/dM)A(a,M,r) = r(r-1)a[(1+\alpha)(r-ra-1)e^{-M} + e^{M(r-1)}],$$

we obtain that R'' satisfies

$$M = \frac{\ln[\alpha(1+R''a-R'')]}{R''}.$$

From the previous expression we obtain that $\lim_{M\to\infty} R'' = 0$, hence there exists $M_1 > 0$ such that for $M > M_1$ we have $R'' < 1$.

We will show that there exists $M_0 > 0$ such that for $M > M_0$ we have $\frac{d^2A(a,M,r)}{dM^2}|_{r=R''} < 0$, thus proving that $R_{a,M}$ is a unimodal function of M, attaining its unique maximum at R''. We have

$$(d^2/dM^2)R_{a,M} = -\frac{(d^2/dM^2)A(a,M,r)}{(d/dr)A(a,M,r)}|_{r=R''},$$

and for $M > M_1$,

$$(d^2/dM^2)A(a,M,r)|_{r=R''} = -R''^2(R''-1)ae^{-M}\alpha(R''-R''a-1) < 0.$$

On the other hand,

$$\begin{aligned}
&(d/dr)A(a,M,r)|_{r=R''} \\
&= (R''-R''a-1)[1+cR''-(1+\alpha)aR''e^{-M}] + (R''-1)(1-a)[1+cR'' \\
&\quad -(1+\alpha)aR''e^{-M}] \\
&\quad +(R''-R''a-1)[c-(1+\alpha)ae^{-M}] + a(1+\alpha)e^{-M}(1+R''a-R'') \\
&\quad +R''aMe^{-M}(1+\alpha)(1+R''a-R'') + 1 - a.
\end{aligned}$$

Using that $\lim_{M\to\infty} R'' = 0$, we obtain

$$\lim_{M\to\infty}(d/dr)A(a, M, R'') = -1 - c < 0,$$

hence there exists M_0 such that for $M > M_0$ there holds

$$\frac{d^2 A(a, M, r)}{dM^2}|_{r=R''_{a,M}} < 0.$$

$\square$

3 Conclusions

For renewal risk models we considered two different types of reinsurance treaties, B and C, respectively, which are combinations of excess-of-loss and quota-share contracts. Following the approach of Centeno [3], in Theorem 1 we obtained conditions on the reinsurance levels a and M and on the premium c, which give for general renewal risk models existence of the corresponding adjustment coefficients $R_{a,M}$.

Due to the complicated form of the equation which satisfies the adjustment coefficient $R_{a,M}$ for general renewal risk models, we considered the particular case of classical risk models with exponentially distributed claims. In case of reinsurance treaty of type B, we obtained in Theorem 2 explicit conditions on a, M, and c, under which the reinsurance adjustment coefficient $R_{a,M}$ is a unimodal function of M. The maximal reinsurance coefficient is the same as the maximal adjustment coefficient obtained by Centeno [3], for a different type of reinsurance treaty, and in this case the ruin probability is minimized among all the reinsurance contracts of type B. Unimodality of $R_{a,M}$ for general renewal risk models with reinsurance treaties of type B remains to be investigated. For classical risk models with exponentially distributed claims and reinsurance treaties of type C, we obtained unimodality or $R_{a,M}$ when $M > M_0$, for some constant M_0, thus minimizing the ruin probability for such reinsurance contracts It remains to study the unimodality of $R_{a,M}$ for reinsurance treaties of type C , for general renewal risk models.

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